The Savings Wedge*

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Abstract

We derive a new criterion for evaluating over- or under-saving in over-lapping generations economies with both ex-ante and ex-post household heterogeneity. Our criterion, which we call the Savings Wedge, nests the traditional 'r vs. g' (Golden Rule) criterion as a special case, but accounts for how aggregate savings affects households' exposure to uninsurable risk and shifts the income distribution, and can be thought of as a risk and redistribution robust Golden Rule criterion. A second-order approximation of the Savings Wedge yields a tractable formula of measurable statistics. Using PSID data from 1978 to 2024, we measure the Savings Wedge in the United States and compare our measure to the traditional Golden Rule. The latter implies mild under-saving for most of our sample as r>g. Relative to the Golden Rule, our criterion suggests substantially more under-saving, driven by both high levels of capital income risk and capital income inequality being greater than labor income inequality.

Keywords: savings, capital accumulation, household heterogeneity, idiosyncratic risk, redistribution

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1 Introduction

What is the optimal amount of aggregate savings? That is, what does it mean for an economy to be over-saving or under-saving, and what would we need to know to make this determination? Knowing whether an economy is over or under-saving is crucial in designing optimal fiscal policy – in particular policies like debt levels and inter-generational transfers that shift the supply of aggregate savings. If we assume that households understand their own private value of savings, then the answer to this question depends on whether there is a wedge between households' private value and the 'social' value of savings.

The benchmark measure of whether an economy is over or under-saving is the *Golden Rule* capital stock [Phelps, 1961]: the level of savings at which investment is equal to the rate of return on capital.[1] Economies with savings rates above the Golden Rule capital stock are *dynamically inefficient*: Saving less would be a Pareto improvement, as it would make the current generation better off without making future generations worse off [Diamond, 1965]. Similarly, the so-called modified Golden Rule criterion allows for discounting the welfare of future generations and states that the optimal gap between the return to capital and investment should increase with this rate of discounting (Cass and Shell [1976]; Dixit [1976]; Atkinson and Sandmo [1980]). In particular, if the economy is growing at constant rate g, then the welfare of future generations can be discounted by (1+g), implying current households should sacrifice less consumption to benefit future (richer) generations. In this case, understanding whether we are saving above or below the Golden Rule level corresponds to the familiar "r versus g" question.[2] Golden Rule criteria capture the trade-off between consuming today and setting resources aside for the future, and have the advantage of being both easy to understand and measure in the data (e.g. Abel et al. [1989]; Barro [2023]).

In this paper, we revisit the question of how to measure over-or under-saving in the context of a model with rich household heterogeneity. In addition to over-lapping generations, our model features uninsurable idiosyncratic shocks to both labor and capital income (ex-post heterogeneity), and permanent labor skill and investment return types (ex-ante heterogeneity). Existing work has shown that models with rich heterogeneity can generate savings behavior that pushes an economy above or below the Golden Rule capital stock.[3] However, both uninsurable risk and systematic inequality *change the welfare consequences of savings*, meaning that the classic Golden Rule may not be the appropriate criterion to

[1] Equivalently, is the marginal product of capital is equal to the depreciation rate? That is, "is the capital sector on net a spout or a sink?" (Abel et al. [1989]).

[2] This question has attracted renewed attention in recent years, see for instance Blanchard [2019] or Barro [2023].

[3] See for example classic studies like Aiyagari [1994] and Angeletos [2007] for ex-post heterogeneity, and more recent work like Morrison [2026] for ex-ante heterogeneity.

evaluate over- or under-accumulation of saving in heterogeneous agent models (Dávila et al. [2012], Krueger et al. [2021]). In this paper, we aim to provide a parsimonious measure of whether an economy is saving the socially optimal amount – analogous to a Golden Rule criterion, but appropriate for heterogeneous agent models.

Specifically, we derive our measure of over- or under- saving directly by simply solving for the marginal value of additional savings from the point of view of a utilitarian planner who internalizes both the distributional consequences and risk externalities associated with greater capital not taken into account by atomistic households. We refer to our measure as the *Savings Wedge*. We show that in a simple over-lapping generations (OLG) model as in Diamond [1965], this wedge corresponds to the modified Golden Rule criterion and implies over (under) saving whenever r is less than (greater than) g.[4] However, in a model with rich heterogeneity, in addition to dynamic trade-offs, our wedge also captures the impact of savings on the *intra-generational* distribution of income, as well as the impact of savings and investment on households' *exposure to idiosyncratic risk*. Intuitively, a greater supply of savings (and therefore a higher capital stock), pushes down rates of return and pushes up wages. This both increases (decreases) aggregate exposure to idiosyncratic labor income (capital income) risk, and redistributes income from capital owners to wage earners. The savings wedge takes these forces into account and can therefore be thought of as a *risk and redistribution robust Golden Rule*.

We then show that our savings wedge can be approximated by a simple formula comprised of a set of measurable statistics. We show that – to second order – the savings wedge is approximately equal to the weighted sum of (i) the variance of both labor and capital productivity shocks, (ii) the distribution of permanent labor and investment income within a generation, and (iii) the distribution of permanent labor and capital income across generations. These 3 terms correspond to the 3 primary components of the savings wedge: uninsurable risk, intra-generational redistribution, and inter-generational redistribution. The weights are intuitive functions of key parameters of the economy like the capital share, the profit share, and the degree of risk aversion, as well as the Pareto weights assigned to types and generations by the planner. We show that our formula's basic structure is highly robust to models with increasing realism and complexity, and take a version of our wedge derived in a rich model to the data to see whether it alters the traditional r vs. g criterion, and why.

Using data from the Panel Study of Income Dynamics (PSID), we measure the savings wedge in the United States from 1978 to 1993 and again from 2007 to 2022.[5] We decompose changes in the savings wedge over time into the contributions of risk and distributional

[4]This holds exactly in the case of log utility.

[5]These are the years in which data on capital income are available.

effects, and compare how our savings wedge criterion compares to the more traditional r vs. g criterion. We also show the sensitivity of our measure of the savings wedge to the Pareto weights assigned to different subgroups and generations.

Our preliminary estimates show that, relative to the traditional Golden Rule criterion which implies that the U.S. was under-saving modestly for most of our sample and slightly over-saving as real rates of return fell in the later years, our criterion suggests a much higher level of under-saving, particularly in the earlier periods. This difference is driven by two forces that the Golden Rule criterion ignores. First, throughout our sample, capital income risk substantially exceeds labor income risk. Since more savings lowers equilibrium rates of return, it reduces households' exposure to capital income shocks while increasing their exposure to labor income shocks. When capital income risk dominates, this shift is welfare-improving, generating a positive risk channel that pushes the savings wedge above the Golden Rule.

Second, throughout our sample, capital income is far more concentrated than labor income: the richest households disproportionately earn their income through capital, while lower-income households — who receive the highest welfare weights under our egalitarian criterion — earn relatively more through labor. Because more capital raises wages and depresses returns, it redistributes income toward labor earners and therefore toward the most highly weighted households. This redistribution motive is absent from the Golden Rule, and its presence substantially amplifies the case for more saving relative to the traditional criterion. As r falls relative to g in the later period, – and therefore the weight on younger households falls – and markups compress the effective labor share, this redistribution motive weakens and eventually reverses, explaining why the gap between our criterion and the Golden Rule narrows toward the end of our sample.

Finally, we test the validity of our approximate savings wedge formula by solving a quantitative model calibrated to match the same steady-state moments used in the formula. The quantitative model also allows us to perform policy counter-factuals. [Quantitative results to be added.]

Framework and methodology. To highlight the key channels driving the savings wedge, we start with a simple overlapping generations (OLG) model with 2 generations, in which households work in a competitive labor market and invest in and operate their own firms as entrepreneurs. We allow for ex-ante heterogeneity in both labor market skill and entrepreneurial productivity. Households also face uninsurable shocks to their labor productivity and entrepreneurial productivity. We use the terms entrepreneurial productivity shocks and capital income shocks interchangeably. In the context of this simple model, we solve the problem of an utilitarian social planner who discounts future generations at a constant rate and can choose the level of savings of all households in the economy, taking the existing distribution of skills and the incompleteness of markets as given. We label the gap between the marginal value of additional savings from the planner's point of view and the marginal value from the point of view of individual households' (their Euler equations) as the *savings wedge.*

Our first result is that the savings wedge can be decomposed into 3 components. The first is a risk exposure channel. By lowering equilibrium rates of return and increasing wages, savings shifts the composition of all households' income away from investment income towards wage income, amplifying the effect of labor productivity shocks and dampening the effects of capital income shocks. The second component is a redistribution term: the sum of each permanent type's share of labor income relative to their share of capital income, weighted by their welfare weight.[6] This term captures the idea that higher wages and lower rates of return redistribute income from capital earners to wage earners. Finally, the third component is the expected welfare weight of future generations, and captures the traditional modified Golden Rule trade-off.

We then take a second order approximation of the wedge around the non-stochastic equilibrium. We show that for small shocks, the wedge is approximately equal to the weighted sum of (i) the variance of the investment income shocks less the variance of labor income shocks, (ii) each type's share of total labor income relative to their share of total capital income within the current generation, and (iii) the share of labor income of future generations. Intuitively, if the variance of capital income shocks is greater than that of labor income shocks, more savings improves welfare by lowering overall exposure to idiosyncratic risk through changes in factor prices. Furthermore, the greater the welfare weight of disproportionate labor earners – in this case those with high Pareto weights, low incomes, or the young – the higher the savings wedge.

Our formula also highlights key *interactions* between the risk and redistribution forces. As capital income inequality grows relative to labor income inequality, this amplifies the redistribution channel, as high-income households tend to be disproportionate capital income earners. However, this dampens the risk channel, as the welfare benefits of lowering exposure to idiosyncratic capital income risk are concentrated among relatively high-income households with lower welfare weights. We show that effect of rising capital income inequality on the savings wedge is therefore ambiguous.

We then solve a richer version of the simple model, adding multiple assets, monopolistic firms, additional fiscal policy, and extend households' life-cycle to an arbitrary number of

[6] Here, a type's welfare weight is their population and Pareto weighted expected marginal utility.

periods, and re-derive the savings wedge in this model. We show that the approximated version of the wedge retains the same structure as before – the weighted sum of the same set of measurable statistics – but the additional model features simply change each term's respective weights. This richer formula can be taken to the data and measured over time. We assume that the idiosyncratic income shocks are log-normally distributed, and use a fixed effect panel regression to estimate the variance of these shocks. At the same time, we estimate each households 'permanent' (capital or labor) income in a given year by estimating average income in a rolling symmetric window around that year. Households are then put into age-specific groups that reflect their labor and capital income percentile.[7] These estimates, along with values for statistics like firm markups, aggregate growth rates, and risk aversion taken from the literature, are plugged into the formula which is plotted over time.

1.1 Related Literature

Our savings wedge nests many forces already identified in the existing literature. For example, the impact of greater savings on aggregate exposure to idiosyncratic labor income risk is the key mechanism in Dávila et al. [2012] and Krueger et al. [2021]. We show that a symmetric logic applies to capital income risk as in Angeletos [2007]. The effect of savings on the distribution of income between households who earn mostly capital income and households who earn mostly labor income also features prominently in Dávila et al. [2012] and Morrison [2026]. Our formula shows what moments in the data determine the relative importance of each of these competing forces, but also reveals key *interactions* between these channels that are obscured in models that consider only 1 or 2 of them at a time.

We are also related to the literature measuring dynamic efficiency in an attempt to assess over or under-saving. Abel et al. [1989] ask if cash flows generated by capital exceed investment in every sector and in every period, implying the economy cannot be dynamically inefficient. Applying this criterion to the United States and six other OECD economies, they find that capital income consistently exceeds investment, and therefore conclude that these countries are not over-saving.

A more recent literature has revisited this question through the lens of the r versus g comparison directly. Blanchard [2019], in his AEA Presidential Address, documents that the US safe rate has been below the nominal growth rate for most of the postwar period and argues that this is more the historical norm than the exception. As a result, the welfare costs of reduced capital accumulation may be smaller than conventionally assumed. In contrast, Barro [2023] argues that the relevant rate for the dynamic efficiency condition is

[7] For example, if a group might consist of all households in the 2nd quintile of labor income and the 3rd quintile of capital income among households in their age group.

the average real return on equity rather than the risk-free rate: while $r^f \leq g$ is common in the data and does not by itself signal dynamic inefficiency, $E(r^e) > E(g)$ is both necessary for dynamic efficiency and consistent with the long-run evidence. Our paper complements this literature by showing that even when r and g are correctly measured, their gap is not a sufficient statistic for evaluating the social value of aggregate savings in a heterogeneous agent economy. Our savings wedge nests the modified Golden Rule as a special case but adds risk and redistribution terms that can independently push the economy toward under- or over-saving.

2 A Modern Heterogeneous Agent Framework

In this section, we solve a simple OLG model in which members of each household both supply labor exogenously to a competitive labor market and invest savings into their private firms as entrepreneurs. Households are grouped into permanent types that govern both their expected labor productivity and entrepreneurial ability. Households also face idiosyncratic uninsurable shocks to both their labor and entrepreneurial productivity. In this setting, we consider the problem of a utilitarian social planner with the ability to dictate savings rates. We define the gap between the marginal value of additional savings to the social planner and the value to individual private households as the savings wedge. We decompose the wedge into three distinct terms corresponding to risk, static redistribution, and dynamic redistribution, and show that the wedge can be approximated by the weighted sum of a set of measurable statistics.

2.1 The Economic Environment

Time is discrete and indexed by $t = 0, 1, \ldots, \infty$. We consider a one-sector production economy with labor and capital inputs. There are two overlapping generations, young y and old o. Households are heterogeneous in two key dimensions: (i) their permanent *ex ante* labor and entrepreneurial ability types, and (ii) their idiosyncratic *ex post* labor and entrepreneurial productivity.

Households. In each period, a new generation of households is born that lives for two periods. Households discount the future period at factor $\beta \in (0, 1)$. Preferences are defined over consumption $(c^y_{j,t}, c^o_{j,t+1})$ with a von Neumann Morgenstern utility function:

$$V_{j,t} = u(c^y_{j,t}) + \beta \mathbb{E}\left[u(c^o_{j,t+1})\right] \ , \quad \text{with} \quad u' > 0, u'' < 0, u''' > 0 \ .$$

Households are grouped into a permanent type, $j \in J$, with corresponding equal discrete measure $\omega_j = \frac{1}{2J}$ where $\sum_j \omega_j = 1$. Households exogenously supply θ_j^y efficiency units of labor to a competitive labor market when young, and θ_j^o when old. When young, households can invest capital into their own private firm. However, households have no other savings vehicle.[8] Type-j households have expected entrepreneurial productivity ζ_j. The household budget constraints are

$$
\begin{aligned}
c_{j,t}^y &= w_t \theta_j^y - k_{j,t+1} \, , \\
c_{j,t+1}^o &= r_{j,t+1} k_{j,t+1} + \epsilon w_{t+1} \theta_j^o \, ,
\end{aligned}
$$

where the general equilibrium wage rates are w_t and w_{t+1}, private firm investment is $k_{j,t+1}$ with corresponding return $r_{j,t+1}$, and ϵ denotes ex-post idiosyncratic labor productivity. We assume that ϵ takes only positive values and $\mathbb{E}[e] = 1$. The households choose $k_{j,t+1}$, $c_{j,t}^y$, and $c_{j,t+1}^o$ to maximize lifetime utility, $V_{j,t}$ subject to their budget constraints.

Technology. The private firm of household type j produces with technology:

$$
y_{j,t} = A_t f(k_{j,t}, n_{j,t}^d) = A_t (\psi \zeta_j k_{j,t})^{\alpha} (n_{j,t}^d)^{1-\alpha} \, .
$$

Here ψ is an idiosyncratic entrepreneurial productivity shock and ζ_j is a type-j household's expected entrepreneurial productivity. Idiosyncratic capital risk, ψ, takes only positive values and $\mathbb{E}[\psi] = 1$. Importantly, ψ is realized at beginning of period $t+1$ *after* capital $k_{j,t+1}$ is installed but *before* employment $n_{j,t+1}^d$ is chosen. Profits are revenues net of labor costs:

$$
\pi_{j,t} = A_t (\psi \zeta_j k_{j,t})^{\alpha} (n_{j,t}^d)^{1-\alpha} - w_t n_{j,t}^d \, .
$$

Consequently, the net return on private savings is $r_{j,t}(\psi, \zeta_j) = \pi_{j,t}/k_{j,t} - \delta$. Entrepreneurs choose their labor demand, $n_{j,t}^d$ to maximize profit, $\pi_{j,t}$ given their invested capital from the previous period, $k_{j,t}$ and the realization of their entrepreneurial productivity shock, ψ.

Technological Growth. We assume A_t grows at a constant exogenous growth rate, g.

Equilibrium. An equilibrium is a sequences of allocations, $\{\{c_{j,t}^y, c_{j,t}^o, y_{j,t}, k_{j,t}, n_{j,t}^d\}_{j \in J}\}_{t \geq 0}$ and prices, $\{w_t\}_{t \geq 0}$ such that labor demand $n_{j,t}^d$ (and therefore $y_{j,t}$) maximizes entrepreneur profit given $k_{j,t}$ and ψ and market wages, w_t, $c_{j,t}^y$, $c_{j,t}^o$ and $k_{j,t}$ maximizes households' lifetime

[8] This assumption on market incompletely is stark, and assumed only to emphasize the role of capital income risk. We relax this assumption in the following section.

utility subject to their budget constraints, and labor and goods markets clear.[9]

Because we have assumed a constant returns to scale production function, and because all entrepreneurs hire labor from a common labor market, individual rates of return can be expressed in terms of the marginal product of aggregate capital. Without heterogeneity in expected entrepreneurial productivity (type based returns), this formulation is exactly the one proposed in Angeletos [2007].

Lemma 1 *Private returns to savings are given by:*

$$r_{j,t} = \frac{\pi_{j,t}}{k_{j,t}} - \delta = \psi \zeta_j r_t - \delta \ , \quad \textit{with} \quad r_t = \alpha k_t^{\alpha-1} n_t^{1-\alpha} \ ,$$

where $k_t = \sum_j \omega_j k_{j,t}$ *is the aggregate capital stock and* $n_t = \sum_J \omega_j(\theta_j^y + \theta_j^o)$ *is the aggregate labor supply. For a proof, see Appendix X.*

Lemma 1 says that returns can be written as the product of entrepreneurs' idiosyncratic investment risk (luck), permanent investment type (group fixed effect), and the marginal product of aggregate capital (time fixed effect). In the absence of type-based returns, the expression in the Lemma 1 corresponds to the idiosyncratic rates of return in Angeletos [2007]. Here, returns depend on aggregate capital – and therefore on the investment decisions of all other entrepreneurs – because all firms hire from the same competitive labor market. Intuitively, by investing more, firms bid up the wage, curbing the returns of other firms.

2.2 The Social Value of Saving

In this section, we consider the marginal value of additional saving to a utilitarian social planner who assigns Pareto weights, λ_j to all type-j households and discounts future generations at constant rate, $\gamma \in [0, 1)$. After substituting households' budget constraints into their utility functions, social welfare can be written as:

$$\mathbb{SW} = \sum_{t=0}^{\infty} \gamma^t \sum_{j \in J} \omega_j \lambda_j \left(u\Big(\theta_j^y w_t - k_{j,t+1}\Big) + \beta \mathbb{E}\Big[u\Big(\psi \zeta_j k_{j,t+1} r_{t+1} + \epsilon \theta_j^o w_{t+1}\Big)\Big] \right) .$$

The total marginal value of additional saving from type-j households born in period t, $\frac{d\mathbb{SW}}{dk_{j,t}}$ is equal to the sum of the *direct* effect on households' private utility plus an *indirect effect*

[9]Note that capital is not traded.

or *wedge*, μ_{t+1} through changes in factor prices.

$$\frac{d\mathbb{SW}}{dk_{j,t+1}} = \gamma^t \omega_j \lambda_j \Big(- u'(c_{j,t}) + \beta r_{t+1} \zeta_j \mathbb{E}\Big[\psi u'(c^o_{ij,t+1})\Big]\Big) + \sum_{\tau=0}^{\infty} \omega_j \zeta_j ((1-\delta)\gamma)^\tau \mu_{t+\tau} \ ,$$

where

$$\mu_{t+\tau} = \left(\sum_J \omega_j \lambda_j \beta \mathbb{E}\left[u'(c^o_{jt+\tau}) \left(\psi \zeta_j k_{j,t+\tau} \frac{\partial r_{t+\tau}}{\partial k_{t+\tau}} + \epsilon_i \theta^o_j \frac{\partial w_{t+\tau}}{\partial k_{t+\tau}} \right) \right] + \gamma \sum_J \omega_j \lambda_j u'(c^y_{j,t+\tau}) \theta^y_j \frac{\partial w_{t+\tau}}{\partial k_{t+\tau}} \right) .$$

The first term corresponds to a type-j households' Euler equation and captures the private marginal value of additional savings. In any decentralized equilibrium, because households are all optimizing, this direct effect of changing savings rates on household welfare is zero. The second term captures the *indirect* effect of the change savings on social welfare. Each type's weight, $\omega_j \zeta_j$ governs the contribution of a type-j households' savings to the aggregate capital stock in period t+1, while the aggregate savings wedge, $\gamma^\tau \mu_{t+\tau}$ captures the welfare impact of a change in the future capital stock on market wages and on average rates of return in period $t+\tau$.[10] Individual households do not internalize these effects. When the savings wedge is positive, more saving and capital would improve social welfare and we say that the economy is under-saving. When this wedge is negative, the economy is over-saving. The following Proposition summarizes the channels through which these changes in wages and average rates of return impact welfare.

Proposition 1 *The savings wedge, μ_{t+1} is proportional to the following.*

$$\mu_{t+\tau} \propto r_{t+\tau} \beta \sum_J \omega_j \lambda_j \Bigg(\underbrace{\mathbb{E}\left[u'(c^o_{j,t+\tau}) \left(\theta^o_j (\epsilon - 1) - \frac{k_{j,t+\tau} \zeta_j}{k_{t+\tau}} (\psi - 1) \right) \right]}_{\textit{idiosyncratic risk}} + \underbrace{\mathbb{E}\left[u'(c^o_{j,t+\tau}) \left(\theta^o_j - \frac{\zeta_j k_{j,t+\tau}}{k_{t+\tau}} \right) \right]}_{\textit{static redistribution}} + \underbrace{\gamma \mathbb{E}[u'(c^o_{j,t+\tau})] \frac{1+r}{(1+g)^{\eta^{RRA}}} \theta^y_j}_{\textit{dynamic redistribution}} \Bigg) \tag{1}$$

Note that along the balanced growth path, $u'(c^y_{j,t+1}) = u'(c^y_{j,t})(1+g)^{-\eta^{RRA}} = \mathbb{E}[u'(c^o_{j,t+1})]\beta(1+r)(1+g)^{-\eta^{RRA}}$. For a proof, see Appendix A.2.

Proposition 1 tells us that changing aggregate savings impacts welfare through three channels. The first term captures the effect of the change in factor prices on households'

[10] With full depreciation ($\delta = 0$), savings today only effects capital tomorrow.

exposure to idiosyncratic risk. Intuitively, an increase in capital generates an increase in the market wage, and an off-setting decline in the aggregate rate of return. For a household whose share of total labor income is equal to their share of total capital income, this shift would have no effect on their expected income in the second period. However, these changes would shift households' exposure to their idiosyncratic labor and entrepreneurial productivity risk. In particular, more capital increases exposure to labor income shocks through higher wages and decreases exposure to entrepreneurial productivity (capital income) shocks. If these shocks are drawn from different distributions, this shift will change the variance and higher-order moments of total income.

The second term captures the effect of the change in factor prices on the distribution of expected income *within the current old generation*. Higher wages and lower average rates of return shifts income towards households who earn a greater share of their total income as a labor income and a smaller share as capital income. If the planner puts greater social welfare weights, $\nu_j \equiv \lambda_j u'(c^o_{j,t})$ on certain household types, then this static redistribution term is increasing in the labor income share relative to the capital income share of these households. Intuitively, if the planner assigns equal Pareto weights to all types – and therefore low-income households are assigned higher social welfare weights – then the static redistribution term will be positive when capital-income inequality is greater than labor income inequality, as low-income types earn a higher share of their household income as wages. If instead the planner sets the Pareto weights to the *Negishi weights* – the weights that rationtionalize the lasseiz-faire distribution of resources across types – then the generalized social welfare weights are uniform and the welfare impact of redistribution is zero.[11]

Finally, the dynamic redistribution term captures the impact of more capital on the wages and consumption of future generations. This term represents the gains from setting aside more resources for the future, and is straightforwardly increasing in the weight assigned to future generations by the planner, γ. Because we assume the economy is on a balanced growth path in which wages growth at constant rate, g, type-j households in the current young generation have marginal utility equal to the marginal utility of the current on in the previous generation, discounted by $(1+g)^{\eta^{RRA}}$. In fact, in the Diamond [1965] model with no risk, ex-ante heterogeneity, or second period labor, this dynamic redistribution term corresponds directly to a modified Golden Rule criterion in a simple OLG model as in Diamond [1965]. This result is summarized in Corollary 1.

[11] In particular, this would imply setting the Pareto weights such that $\frac{\lambda_j}{\lambda_{j'}} = \frac{u'(c^o_{j'})}{u'(c^o_j)}$.

Corollary 1 *Suppose ϵ and ψ are constants, $J = 1$, and $\theta^o = 0$. Then,*

$$\mu_{t+\tau} \geq 0 \text{ iff. } \frac{1}{\gamma}(1+g)^{\eta^{RRA}} \leq 1 + r_{t+\tau} = 1 + f_k - \delta \; .$$

For a proof, see Appendix A.2.2.

From Corollary 1 we see that, when household heterogeneity is removed, the savings wedge captures precisely the same dynamic trade-offs that generates the Golden Rule capital stock. As γ approaches 1, whether $\mu_{t+\tau}$ is positive or negative – and therefore whether the economy is over or under-saving – collapses to a dynamic efficiency, "r versus g" criterion. In particular, if $\gamma = 1$ (no discounting of future generations) and we assume log utility, then $\mu_{t+\tau} > 0$ if and only if $r > g$. By comparing the savings wedge in Proposition 1 and Corollary 1, we can see that the savings wedge nests the classic Golden Rule criterion, but captures additional welfare impacts of savings generated by systematic labor and capital income inequality and idiosyncratic risk. For this reason, our savings wedge can be thought of as a *risk and redistribution robust Golden Rule.*

2.3 Discussion

Relationship to related literature. Proposition 1 shows how our framework helps unify the existing literature on the over- or under- accumulation of savings and capital. Dávila et al. [2012] study the normative properties of a model with infinitely lived ex-ante identical households and uninsurable idiosyncratic labor income shocks as in Aiyagari [1994]. As households accumulate shock realizations, their start of period wealth begins to diverge. To show the effect of this divergence, in Section 2.2 of their paper, the authors consider a 2-period version of their model with initial heterogeneity in wealth. They show that whether the economy is over- or under- saving depends on the strength of 2 opposing forces: a 'pecuniary externality' in which more capital and higher wages amplifies idiosyncratic risk, and a redistributive force that redistributed from the wealthy to the non-wealthy.

These forces are nested by our savings wedge, and correspond to a case in which $\gamma = 0$, $\psi = 1$ is a constant, and $\theta_j = 1$ for all j. That is, a case without weight on future generations, without capital income risk, and without ex-ante labor productivity heterogeneity, implying wealth inequality is more severe than labor income inequality by construction.

In addition to nesting the forces operating in Dávila et al. [2012], our framework – which allows for ex-ante return heterogneity – clarifies that what matters for the strength of the redistribution term is the degree of labor income inequality relative to *capital income* inequality rather than wealth inequality per se. This is an important distinction given that the degree of capital income inequality appears to be more severe than wealth inequality in the data [Gaillard et al., 2023].

Both Panousi and Reis [2021] and Park [2013] conduct normative policy analysis in a model with idiosyncratic investment income risk as in Angeletos [2007] The model studied in Angeletos [2007] and Panousi and Reis [2021] is nested by our savings wedge, and corresponds to a case in which ζ_j and θ_j are uniform, ϵ is a constant, and $\gamma = 0$. That is, a model without ex-ante heterogeneity, labor income risk, or weight on future generations.[12] Angeletos [2007] answers a *positive* question: does investment income risk generate more or less saving relative to an economy without it. The normative policy exercise conducted in Panousi and Reis [2021] is closer in spirit to our exercise – their optimal capital tax is meant to address the over or under accumulation of savings – but whether the optimal net tax is positive or negative depends both on whether more savings would be welfare improving, and on whether a higher tax would generate more or less saving.[13] In contrast, our framework makes clear that, all else equal, idiosyncratic income risk leads to under-saving.[14]

Finally, our framework nests the literature on over- and under- accumulation of savings in OLG models. As shown in Corollary 1, in the absence of idiosyncratic risk and ex-ante heterogeneity, our model nests the classic Golden Rule criterion emphasized in Diamond [1965]. Our model also nests the trade-offs present in Krueger et al. [2021] who study optimal capital taxation in a model with idiosyncratic labor income risk and over-lapping generations. Their benchmark model corresponds to a case in which $\gamma > 0$, but ϵ is a constant, and both θ_j and ζ_j are uniform.

One important takeaway from Proposition 1 is that whether an economy is over or under-accumulating savings depends crucially on both the nature of the income shocks households face and on their joint distribution. Krueger et al. [2021] consider an extension with both labor and capital income risk, but do not explore interactions between ex-ante heterogeneity and risk or explicitly link the relative strengths of labor or capital income risk to moments of the joint distribution of shocks. Furthermore, both Angeletos [2007] and Panousi and Reis [2021] study the effect of idiosyncratic capital income risk in the presence of *additive* income endowment risk. When aggregate capital increases, this dampens exposure to capital income risk as average rates of return fall, but the subsequent rise in wages does not amplify exposure to the endowment risk. In contrast, labor income risk in our model is *multiplicative*, meaning that the welfare effects of the two types of risk push in opposite directions and could

[12] It should be noted that both papers include a riskless asset. We show in the next Section that the presence of a riskless asset does not qualitatively change our results.

[13] That is, whether the income or substitution effects dominate.

[14] These papers have both a risky and riskless asset. In the following section, we re-derive our formula in the presence of a multiple riskless assets and show that it is qualitatively the same.

cancel one another out.

The advantage of our framework is that by unifying many studies int the existing literature into a single formula, we are able to both show how features of the economic environment amplify or dampen each competing channel and to uncover key interactions between ex-ante inequality and idiosyncratic risk. In particular, in Section 2.5 we examine how the effect of rising capital income inequality – relative to labor income inequality – on the savings wedge depends crucially on the relative severity of capital income risk compared to labor income risk. Furthermore, our measurable formula will allow us to estimate the degree of over- and under- saving empirically.[15]

Efficiency vs. Redistribution. A non-zero savings wedge tells us that the level of capital may both be *inefficient* and may generate a distribution of income that is *too unequal relative to the planner's preferences.* The presence of idiosyncratic risk is the source of the inefficiency. By failing to internalize the effect of saving on factor prices, households impose a pecuniary externality on other households. In particular, when capital income risk is higher (lower) than labor income risk, savers fail to internalize that by saving more (less), they could reduce aggregate exposure to risk.

The second two terms capture the planner's pure redistribution motive. In other words, it is possible to find Pareto weights $\{\{\lambda_j\}_{j\in J}, \gamma\}$ that *rationalize* any distribution of expected income such that both the static and dynamic redistribution terms are 0.[16] In the absence of idiosyncratic risk (when both ϵ and ψ are constants), the savings wedge simply captures the deviation of the Negishi weights that rationalize the equilibrium and the planner's preferred Pareto weights.

2.4 The Measurable Savings Wedge

In order to evaluate whether an economy is over- or under- saving empirically, we take a second-order approximation of the savings wedge around the zero-risk equilibrium. Doing so transforms the expression for $\mu_{t+\tau}$ from Proposition 1 into a formula of measurable statistics and welfare weights that can be taken to the data. Furthermore, we can use our measurable statistic formula to examine what features of the economic environment amplify and dampen each component of the wedge. In Section XX, we re-derive the measurable saving wedge in the context of a more realistic model, and show that the basic structure of our measurable formula is highly robust to additional realism. Before presenting the formula, we first must

[15]Want this exercise to be in the same spirit as Abel Mankiw.

[16]The Negishi or market weights.

define the coefficients of relative risk aversion and relative prudence, as well as our generalized social welfare weights.[17]

Definition. Define the coefficients of relative risk aversion (η^{RRA}) and relative prudence (η^{RPR}) as follows (see Kimball [1990]):

$$\eta^{RRA} \equiv -\mathbb{E}\left[c_j^o\right] \frac{u''(\mathbb{E}\left[c_j^o\right])}{u'(\mathbb{E}\left[c_j^o\right])} > 0 \; ,$$
$$\eta^{RPR} \equiv -\mathbb{E}\left[c_j^o\right] \frac{u'''(\mathbb{E}\left[c_j^o\right])}{u''(\mathbb{E}\left[c_j^o\right])} > 0 \; .$$

Definition. Define the general social welfare weight for type-j households $\nu_j^o \equiv \lambda_j \mathbb{E}[u'(c_{j,t+\tau}^o)]$.

Along with the planner's discount factor on future generations, these terms are crucial inputs into the formula that determine the importance of risk and redistribution for social welfare.

Proposition 2 *Suppose that the $Cov(\psi, \epsilon) = 0$. Denote aggregate output in period $t+\tau$ as $Y_{t+\tau}$. Then the second-order approximation of the savings wedge around the zero-risk equilibrium is given by the following expression.*

$$\mu_{t+\tau} \approx (1-\alpha) r_{t+\tau} \beta \left[\mu_I + \mu_R + \mu_D \right] \; ,$$

where the sub-components are given by:

$$\mu_I = \eta^{RRA} \sum_j \nu_j^o \frac{Y_{t+\tau}}{\mathbb{E}\left[c_{j,t+\tau}^o\right]} \times \left(\alpha \omega_j \left(\frac{\zeta_j k_{j,t+\tau}}{k_{t+\tau}} \right)^2 Var(\psi) - (1-\alpha) \omega_j (\theta_j^o)^2 \, Var(\epsilon) \right) \; ,$$
$$\mu_R = \sum_j \nu_j^o \left(\omega_j \theta_j^o - \omega_j \zeta_j \frac{k_{j,t+\tau}}{k_{t+\tau}} \right) \left(1 + \frac{1}{2} \eta^{RRA} \eta^{RPR} \frac{Var(c_{j,t+\tau}^o)}{\mathbb{E}\left[c_{j,t+\tau}^o\right]^2} \right) \; ,$$
$$\mu_D = \gamma \sum_j \omega_j \nu_j^o (1 + r_{t+\tau})(1+g)^{-\eta^{RRA}} \theta_j^y \; .$$

For a proof and to see a complete statement of the Proposition with $Cov(\psi, \epsilon) \neq 0$, see Appendix A.2.1.

The first term corresponds to the idiosyncratic risk channel in Proposition 1. As relative risk aversion increases, the impact of shifting exposure to idiosyncratic risk on the savings

[17] Cite Saez and Stantcheva for this.

wedge is amplified. This term is increasing in the variance of the capital income shocks and decreasing in the variance of labor income shocks. As discussed in Proposition 1, by increasing wages and decreasing average returns, savings increases households' exposure to labor income risk and decreases exposure to capital income risk. Now however, our measurable formula tells us that what determines the relative strength of these two channels (to second order) is the *weighted* variance of the two shocks.

As the labor share, $1-\alpha$ grows, households earn a lower share of total income through capital and a greater share through labor, meaning that the welfare gains associated with decreasing capital income risk exposure fall, while the welfare costs of increasing exposure to labor income risk increase. All else equal, an increase in the labor share implies a smaller savings wedge. The relative strength of the two income risks channels also depends on the joint distribution of capital and labor income. The planner puts a higher weight on the welfare of household types with a high social welfare weight, ν_j and a low share of total income, $\frac{\mathbb{E}[c^o_{j,t+\tau}]}{Y_{t+1}}$. The higher the capital income share, $\frac{\zeta_j k_{j,t+\tau}}{k_{t+1}}$ of highly weighted households, the greater the importance of the capital income risk channel. Similarly, the higher their share of total labor income, θ^o_j, the greater the importance of the labor income risk channel.

The second and third term correspond to the static redistribution and dynamic redistribution terms in Proposition 1. Again, more savings and higher capital constitutes a redistribution towards households who earn relatively more of their income in the labor market rather than through capital investment. The higher the relative social welfare weight of those types, the greater the welfare impact of this redistribution term. Similarly, the greater the weight put on future generations, the greater the welfare impact of savings.

From the formula we can see the importance of both relative risk aversion and relative prudence in determining the degree of over or under-saving. As relative risk aversion increases, the size of the idiosyncratic risk channel, μ_I increases. At the same time, the product of η^{RRA} and η^{RPR} amplifies the contribution of the welfare of type-j households to the redistribution term. The product $\eta^{RRA}\eta^{RPR}$ captures both how much households dislike risk (η^{RRA}) and how the curvature of their marginal utility responds to uncertainty (η^{RPR}). Households facing more volatility relative to their expected consumption get proportionally larger welfare weights in the redistribution term, μ_R.

2.5 Increasing Capital Income Inequality Relative to Labor Income Inequality

The measurable formula presented in Proposition 2 can help us understand how shifts in the economic environment over time change the relative magnitude of each of the components of the savings wedge. One notable change in the United States in recent decades has been the increase in the severity of *capital income inequality relative to labor income inequality*. We can use our formula to ask how this shift affects the degree of over- or under-saving.[18]

Consider the effect of a small increase in capital income inequality: a redistribution of capital income shares away from household types with lower-than-average shares of capital income towards households with a higher-than-average share, keeping aggregate capital and the distribution of labor productivity unchanged. Below, we show that the impact of such a change on the savings wedge depends on (1) the magnitude of capital income risk relative to labor income risk, and (2) the current degree of *relative* income inequality; here we define relative income inequality as the covariance between households' total income (and therefore their consumption) and their share of total capital income relative to their share of total labor income. This term captures the severity of capital income inequality relative to labor income inequality – when this term is high, 'rich' households have a disproportionately higher share of total capital income. [19]

We first consider the effect of an increase in capital income inequality on the idiosyncratic risk exposure term, μ_I. Intuitively, this term is the welfare weighted-sum of households' net exposure to risk. Receiving a greater share of capital income simultaneously lowers a household's welfare weight (by increasing their expected consumption), but also concentrates exposure to capital income risk relative to labor income risk (a concentration effect). Which of these two forces dominates determines the impact of an increase in capital income inequality on μ_I.

In the absence of *relative* inequality, each type's share of capital income is the same as their share of labor income. As a result, all households have the same net-exposure to idiosyncratic risk: a change in factor prices would increase the variance of all type's income in a proportional way. Given this even exposure to risk across types, the overall importance of the idiosyncratic risk channel is determined by the severity of *total income* inequality. When total income inequality is higher – that is, the variance of consumption is greater – the consequences of changing households' exposure to risk are amplified. Through a *reweighting* effect, an increase in capital income inequality increases the degree of total consumption inequality, and amplifies the benefits (costs) of greater capital when the variance of ψ is greater (smaller) than the variance of ϵ.

If at the same time there was already substantial *consumption* inequality before the change, more capital income inequality would unambiguously make μ_I more negative, as it

[18]One notatble study of the rise of capital income inequality relative to labor income inequality in the United States since xxx is... (cite)

[19]A zero covariance (no relative inequality) implies that $\frac{\zeta_j k_{j,t+1}}{k_{t+1}} = \theta_j$ for all j. A high correlation implies that households with high over-all income are disproportionate capital income earners.

would further concentrate net risk exposure among households with lower incomes – and therefore higher welfare weights. Intuitively, the benefits of reducing the exposure to capital income risk are concentrated among households whom the planner now 'cares less about'. Whenever $\eta < 2/\alpha - 1$, the absolute magnitude of this concentration effect is greater than the reweighting effect. These results are summarized in Proposition 3.

Proposition 3 *Suppose $\lambda_j = 1$ for $j \in J$, $\eta < (2/\alpha - 1)$, and $Var(\mathbb{E}[c^o_{j,t+1}]) > 0$. Define $\mathcal{X}^H \subset J$ as the set of all types with higher than average capital income. Consider a small change in the distribution of capital income, $\{\Delta_j\}_{j \in J}$ such that $\sum_j \Delta_j = 0$ and $\Delta_j > 0$ for all $j \in \mathcal{X}^H$ and $\Delta_j < 0$ for all $j \notin \mathcal{X}^H$.*

If $Cov\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}\theta_j}, \mathbb{E}[c^o_{j,t+1}]\right) = 0$, then the total change in the idiosyncratic risk term, $d\mu_I < 0$. For a proof, see Appendix A.3.

Intuitively, Proposition 3 tells us that for an *egalitarian planner* facing *no relative inequality*, as long as the concentration effect outweighs the reweighting effect, an increase in capital income inequality always decreases the idiosyncratic risk channel. If however, relative inequality were high – and therefore capital income risk was already heavily concentrated among a small subset of households – further concentrating this risk would be costly to the planner.[20] For sufficiently high levels of capital income risk, this would imply that the planner would put *more* welfare weight on these high-risk-exposure types, and therefore assign greater importance to reducing capital income risk. As a result, the effect on μ_I of greater capital income inequality would be ambigous.

We can also use our measurable formula to examine the impact of the increase in capital income inequality described above on the redistribution term, μ_R. Assuming an egalitarian planners facing some overall income inequality, the effect of increasing capital income inequality relative to labor income inequality on the redistribution channel is summarized in the following proposition.

Proposition 4 *Suppose $\lambda_j = 1$ for $j \in J$ and the $Var(\mathbb{E}[c^o_{j,t+1}]) > 0$. Define $\mathcal{X}^H \subset J$ as the set of all types with higher than average income. Consider a small change in the distribution of capital income, $\{\Delta_j\}_{j \in J}$ such that $\sum_j \Delta_j = 0$ and $\Delta_j > 0$ for all $j \in \mathcal{X}^H$ and $\Delta_j < 0$ for all $j \notin \mathcal{X}^H$.*

If any of the following conditions are satisfied:

[20] That is, the increase in expected income and consumption from an increasing capital share is not worth the increased variance of income.

(i) $Cov\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}\theta_j}, \mathbb{E}[c^o_{j,t+1}]\right) = 0$,

(ii) $Cov\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}\theta_j}, \mathbb{E}[c^o_{j,t+1}]\right) > 0$ *and* $Var(\psi) < \frac{1-\alpha}{\alpha} Var(\epsilon)$,

(iii) $Cov\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}\theta_j}, \mathbb{E}[c^o_{j,t+1}]\right) < 0$ *and* $Var(\psi) > \frac{1-\alpha}{\alpha} Var(\epsilon)$

then the change in the redistribution term, $d\mu_R > 0$.

For a proof, see Appendix A.3.

Proposition 4 provides sufficient conditions under which increasing capital income inequality increases the redistributive effect of aggregate savings. In the absence of relative inequality, all types have the same net exposure to idiosyncratic risk and therefore an increase in capital income inequality relative to labor income inequality unambiguously amplifies the redistribution effect. When relative inequality is positive – the rich are disproportionate capital income earners – but labor income risk is more severe than capital income risk, lower income households have higher net risk exposure. The same is true when relative inequality is negative and capital income risk is more severe.

In all three cases, households with lower expected income also have income that is at least as variable as that of higher income households. As a result, the planner buts unambiguously higher weight on these households. Greater capital income inequality then implies that aggregate savings redistributes a greater amount from low-welfare weight types towards high welfare weight types, increasing μ_R.

Note that the conditions outlined in Proposition 4 are sufficient but not necessary. Even if high-income households have greater net risk exposure – as would be the case if capital income risk were more severe and high income households were disproportionate capital income earners – it may still be the case that increasing capital income inequality relative to labor income inequality amplifies the redistribution channel. However, for sufficiently high levels of capital income risk, further concentrating capital income among high income types may actually dampen μ_R, as the high variance of these households' income would lead the planner to weigh these households more.

3 A More General Model

In this Section, we re-derive the savings wedge in a more general setting with additional savings instruments, a general life-cycle of arbitrary length, fiscal policy, and market power.

Doing so gives us a measurable formula whose components have closer analogs in the data. We show that, even in a more general environment, the savings wedge is driven by the same three forces: idiosyncratic risk, static redistribution, and dynamic redistribution, and that the measurable savings wedge is simply a re-weighted version of the formula presented in Proposition 2.

3.1 The Economic Environment

Households. In each period, a new generation of households is born that lives for H periods. Preferences are defined over consumption at each age $h \in \{1, 2, ..., H\}$, $(c^h_{j,t}$ with a von Neumann Morgenstern utility function:

$$V_{j,t} = \sum_{h=0}^{A} \beta^h u(c^h_{j,t+h}) \ , \quad \text{with} \quad u' > 0, u'' < 0, u''' > 0 \ .$$

Households at age h exogenously supply θ^h_j efficiency units of labor to a competitive labor market. In addition to investing capital, $k^h_{j,t+h}$ into their own private firm, households can invest capital, $k^{ch}_{j,t+h}$ into a public corporate sector through a financial intermediary and can save and borrow in a one period bond, $b^h_{j,t+h}$. Households are subject to a linear labor tax, $\tau_{\ell,t}$ on their labor income and a capital tax, $\tau_{k,t}$ on their capital income. Households' period budget constraint at age h is now given by:

$$\begin{aligned} c^h_{j,t+h} = (1-\tau_{\ell,t+h})\theta^h_j \epsilon w_{t+h} - b^h_{j,t+h+1} - k^{ch}_{j,t+h+1} - k^h_{j,t+h+1} + (1-\tau_{k,t+h})((1+r^b_{t+h})b^h_{j,t+h} \\ + (1+r^c_{t+h})k^{ch}_{j,t+h} + (1+r_{j,t+h})k^h_{j,t+h}) \ . \end{aligned}$$

Final Good Firm. A competitive final good firm aggregates goods from intermediate goods firms who produce differentiated varieties $i \in [0, 1]$ with the following CES production function.

$$y_t = \left(\int_I y_{it}^{\frac{\rho-1}{\rho}} di \right)^{\frac{\rho}{\rho-1}} \ .$$

Intermediate Goods Firms. Let A_t be aggregate productivity. Both private firms run by entrepreneurs and the corporate sector produce intermediate goods to sell to the final goods aggregator using both labor and capital. Entrepreneurs investing $k_{j,t}$ into their private business and earn the private stochastic return $r_{j,t}$, which we derive below. Entrepreneurs producing variety-i operate the following production function with productivity shock, ψ_{ijt}.

$$y_{ij,t} = A_t(\psi\nu_j)^\alpha(k_{ij,t})^\alpha(\ell^d_{ij,t})^{1-\alpha} \ .$$

A competitive intermediary pools capital invested into the corporate sector one period in advance and invests it in corporate firms. These firms produce variety i using labor and capital according to the following production function:

$$y^c_{i,t} = A_t(k^c_{i,t})^\alpha(\ell^d_t)^{1-\alpha} \ .$$

After corporate firms produce, they return the un-depreciated capital to the competitive intermediary who transfers the un-depreciated capital to households along with a share of corporate firm profits proportional to their investment, $k^c_{j,t}$.

Technology growth. As before, we assume that aggregate productivity grows at constant exogenous rate, g.

Government. The government uses tax revenue to pay interest on its debt, B_t and to fund government spending, G_t, i.e.:

$$G_t + r^b_t B_{t-1} = \tau_{kt}(b_t r^b_t + \sum_J \omega_j k_{j,t} r_{j,t} + k^c_t r^c_t) + \tau_{\ell t} w_t + B_t \ . \tag{2}$$

Equilibrium Definition. To be added.

Definition. We define aggregate capital, k_t as follows:

$$k_t = \sum_j \omega_j \nu_j k_{j,t} + k^c_t \ .$$

Lemma 2 (Returns and Wages) *Private returns to savings are given by:*

$$r_{j,t} = \frac{\pi_{j,t}}{k_{j,t}} = \psi\zeta_j r_t, \quad \textit{where } r_t = \alpha k_t^{\alpha-1}\ell_t^{1-\alpha} \ .$$

Corporate returns, $r^c_t = r_t$ *and real market wages,* $w_t = \frac{\rho-1}{\rho}(1-\alpha)k^\alpha_t \ell^{-\alpha}_t$ *For a proof, see Appendix A.4.1*

From Lemma 2 we can see that monopolistic competition drives a wedge between wages and the aggregate marginal product of capital. Note however that because households manage their private firms, the markup does not appear in the rate of return on capital. Corporate capital and bonds both return the same risk-free rate of return.

3.2 A Measurable Savings Wedge

As before, we consider the marginal value of additional saving to our egalitarian social planner and again define our savings wedge as the gap between households' private value and the social value of savings. The new planning problem is given by equation XX.

Proposition 5 *Assume the economy is currently on the balanced growth path. Define the auxiliary welfare weights defining* $\nu_{j,t} \equiv \lambda_j u'(\mathbb{E}[c^H_{j,t}])\frac{y}{\mathbb{E}[c^H_j]}$ *and* $\hat{\nu}^h_{j,t} \equiv \nu_{j,t}\left(1+\frac{1}{2}\eta^{RRA}\eta^{RPR}\frac{Var(c^h_{j,t})}{\mathbb{E}[c^h_{j,t}]^2}\right)$, *and denote the inverse of the monopolists' markup,* $\mathcal{M} \equiv \frac{\rho-1}{\rho}$. *Then the second-order approximation of the savings wedge around the zero-risk equilibrium is given by the following expression:*

$$\mu \propto r\left(\mu_I + \mu_R\right),$$

where

$$\mu_I = \sum_j \nu_j \sum_h \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}}\right)^{H-h}\left(\alpha\omega_j\, Var(\psi)\left(\frac{\zeta_j k^h_j(1-\tau_k)}{k}\right)^2 - (1-\alpha)\omega_j\, Var(\epsilon)\left(\theta^h_j(1-\tau_\ell)\mathcal{M}\right)^2\right.$$
$$\left. - \left(\mathcal{M}\alpha + \alpha - 1\right)(1-\tau_\ell)(1-\tau_k)\omega_j\theta^h_j\frac{\zeta_j k^h_h}{k}\,Cov(\epsilon,\psi)\right),$$
$$\mu_R = \sum_j\sum_h \hat{\nu}^h_j\left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}}\right)^{H-h}\left(\omega_j\theta^h_j(1-\tau_\ell)\mathcal{M} - \left(\frac{\omega_j\zeta_j k^h_j}{k} + \frac{\omega_j k^{hc}_j}{k} + \frac{\omega_j b}{k}\right)(1-\tau_k)\right).$$

For a proof, see Appendix A.2.1.

From Proposition 5, we see that the measurable savings wedge maintains the same basic structure as in the simple model – an idiosyncratic risk term and a redistribution term – but with richer weights. The idiosyncratic risk term now weights the variance of capital income shocks by the welfare-weighted sum of *after-tax* capital income shares for all types *across generations*. Similarly, the variance of labor income is weighted by the sum across types and generations of after-tax shares of labor income, *discounted by the markup.* Intuitively, because monopolistic pricing generates a wedge between the market wage and worker's marginal product, dampening the impact of addition capital on wages. We also allow for a non-zero covariance between households' labor and capital income.

The redistribution term is now the welfare-weighted sum across types and generations of households' after-tax labor income share, discounted by the markup, less their total after tax capital income share. Households' capital income share includes fixed income and public capital income alongside private capital income. Note that we have consolidated the static and dynamic redistribution terms into a single term accounting for both intra- and inter-generational redistribution.

Corollary 2 *Suppose there is no idiosyncratic risk, taxes, markups, and no ex-ante heterogeneity within generations. Then the measurable savings wedge simplifies to:*

$$\mu \propto r \cdot \mu_R,$$

where

$$\mu_R = \sum_h \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}} \right)^{H-h} [\Theta_h - Z_h], \tag{3}$$

and where $\Theta_h \equiv \sum_j \omega_j \theta_j^h$ *and* $Z_h \equiv \sum_j \frac{\omega_j \zeta_j k_j^h}{k}$ *denote the total population-weighted labor and capital income shares of generation* h*, respectively.*

Equation (3) is reminiscent of Corollary 1, which shows that without risk or ex-ante heterogeneity, assuming that the young work and the old live off capital income, the savings wedge collapses to an r vs. g criterion. Here, if we restricted $h \in (0,1)$ and allocate all labor income to the young and all capital income to the old, we recover this criterion.

In the following section, we take the measurable formula in Proposition 5 as well as the simplified formula from equation (3) to the data to see how the savings wedge has evolved over time, to account for the forces driving these changes, and to compare our criterion to the traditional r vs. g measure.

4 Measurement

In this section, we measure the savings wedge over time using a combination of existing estimates from the literature and our own estimates.

A central empirical question in measuring the savings wedge is the appropriate measure of the return to capital, r. We consider two alternative measures. The first is the (annualized) real yield on 10-year US Treasury bills as in Blanchard [2019]. The second is the average real rate of return on equity. As documented in Barro [2024], these three measures can diverge

Table 1: Statistics from the Literature

Statistic	Description	Value	Source
η^{RPR}	Coeff. relative prudence	$1+\eta^{RRA}$	Kimball [1990]
η^{RRA}	Coeff. relative risk aversion	1–5	Chetty [2006], Barsky et al. [1997]
$\mathcal{M}$	Inverse markup		De Loecker et al. [2020]
τ_k	Capital income tax rate	0.4	Mendoza et al. [1994]
τ_ℓ	Labor income tax rate	0.3	Mendoza et al. [1994]
r	Return to capital		Shiller [2026][21]
α	Capital share	.33	
g	Growth rate		BEA NIPA

substantially — particularly in recent decades as the bond rate has fallen while average returns to capital have remained relatively stable — implying that the dynamic efficiency assessment is sensitive to which concept of r one adopts. We use the 10-year bond rate as our baseline, but present results under both measures and discuss the sensitivity of the savings wedge to this choice.

Our data come from the Panel Survey of Income Dynamics (PSID), a longitudinal panel survey of several thousand US households and their descendants starting in 1968. The data include information on household labor income as well as household holdings of and income from various types of assets. We include all households in our sample, with no additional sample selection criteria. For each variable of interest, we use the PSID computed household aggregate if it exists, or otherwise construct the variable as the sum of head and spouse sub-measures. As the PSID questionnaire varies over time, certain measures of income and wealth are not available for the entire duration of the panel. In particular, wealth information is only available in 1984, 1989, 1994, and 1999 onward, expenditure information is only available from 1999 onward, and asset income is missing between 1995 and 2003. All measures are transformed into 2017 USD using the PCE price index and we apply no additional truncation or winsorization.

Household labor income is constructed as the sum of head and spouse labor income, excluding business income. Asset income is constructed as the sum of head and spouse farm, business, rent, interest, dividend, room and board, trust and royalty, market gardening, and other asset income. We define total capital income as aggregate asset income, and the *risky* component as business income specifically. Transfer income is constructed as the sum of social security income and all transfers excluding social security. Wealth is taken directly from household net wealth. Consumption expenditure is constructed as total expenditure less flow payments for mortgages and property taxes.

For the years from 2003 onward, both total capital income and its business income component are directly observable in the PSID, allowing us to compute each age-type j group's share of business income relative to total capital income. For earlier years in the sample, the PSID does not separately identify business income. As a result, we cannot estimate the variance of idiosyncratic business income shocks. In these years therefore, we impute risky capital income *shares* for each age-type j group by multiplying the group's total capital income by its average business income share estimated from the later sample and fix the value for the variance of ψ at it's initial value in the later sample. Table 2 presents the sampling-weight adjusted summary stats for the full sample, as well as the first and last year labor and asset income are jointly available.

Table 2: PSID Summary Statistics

	Full Sample	1976	2023
HH Labor Income	47,981.55	37,959.98	58,733.16
HH Asset Income	5,503.73	4,017.68	4,838.18
HH Asset Income (Risky)	2,734.81	–	1,839.77
HH Transfer Income	9,050.05	5,194.83	13,801.45
HH Consumption	31,087.75	–	35,395.88
HH Wealth	361,859.21	–	473,435.31
Head — Age	48.84	45.80	53.39
Head — Male	0.70	0.72	0.68
Head — Years of Schooling	12.90	11.67	13.73
HH — Married	0.50	–	0.43
HH — Number of Children	0.68	0.92	0.49
Observations	306,002	5,861	9,152

4.1 Empirical Strategy

Our first objective is to isolate the 'permanent' component of a households' labor and capital income in order to group them by permanent income types.[22] In order to do this, we follow Straub (2019) and construct symmetric moving averages for each individual household for each type of income. In particular, for individual i, income type y, age h, time t, and window W, we define:

[22] This corresponds to ex-ante types, j, in our model.

$$\bar{y}^{ih}_{t,W} = \sum_{\tau=t-\frac{W-1}{2}}^{t+\frac{W-1}{2}} y^{i,h+\tau}_{\tau} \tag{4}$$

We construct these symmetric averages using both $W = 5$ and $W = 9$ for years in which both labor and asset income are jointly present: 1976 to 1993 and 2005–2023. The measure is only constructed for observations in which data are available for the entire window. Note that since the PSID switched from annual to biennial in 1999, the same window length contains fewer observations in the later period than the earlier one. We refer to $\bar{y}^{ih}_{t,W}$ as a household's permanent labor income when y is labor income, and their permanent capital income when y is capital income.

We then use these symmetric averages to group households into permanent income groups depending on which percentile their permanent capital and labor income falls into for households of their age in their year. In particular, we calculate quintiles of both types of income for a particular year for each 10 year age group between ages 25 and 65. An income group for households in a particular age group in a particular year is a permanent labor income quintile–permanent capital income quintile pair, with the bottom 2 capital income quintiles grouped together. Because households who are older than 65 are less likely to work, we maintain the same 4 capital income groups for the 65 to 74-year-old group and the 75 and older age-group, but collapse the labor income groups into 3 groups: those with no labor income, and those with above or below median labor earnings for over-65-year-olds in that year.

Once households are assigned to permanent income groups for their age and year, we calculate several key statistics for each group: in particular, we calculate each group's (i) population share, (ii) share of total labor income, (iii) 'risky' capital income as a share of total capital income, and (iv) total income shares. These shares correspond to the statistics in Table 1 where we use total income shares as a proxy for consumption shares.[23,24]

Figure 1 plots the evolution of labor income shares over time for each permanent income quintile, while Figure 2 plots the evolution of capital income shares. From the figures, it is clear that both substantial labor income and capital income inequality are present in the data, and that the share of the highest quintiles rises in the later years of our sample. However, there is substantially more capital income inequality as measure by the top quintiles share of total income. In the period between 1978 and 1993, the top quintile controlled around 80 percent of total capital income, rising to 90 percent in the years after 2005.

[23] This exercise is analogous to the one performed in Kekre and Lenel [2025].

[24] For the years after 2005, data on consumption is directly available and we calculate consumption shares for each group.

Figure 1: Labor income shares by permanent labor income quintile

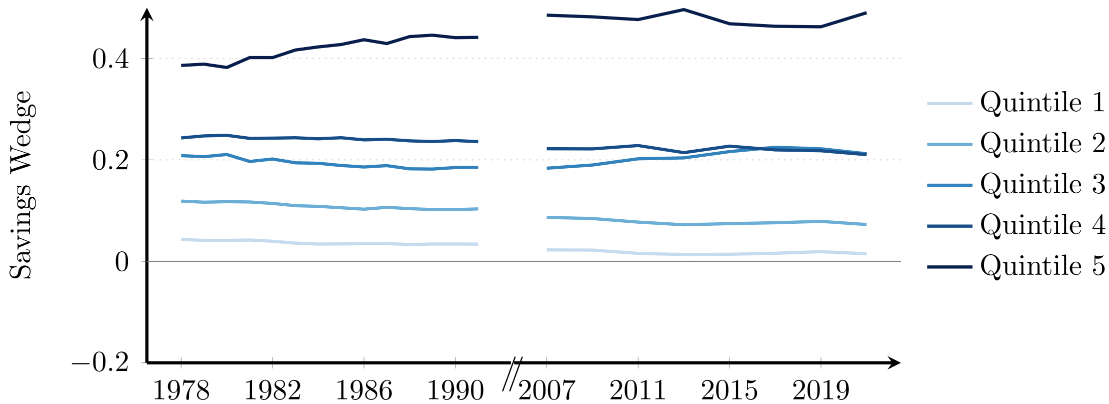
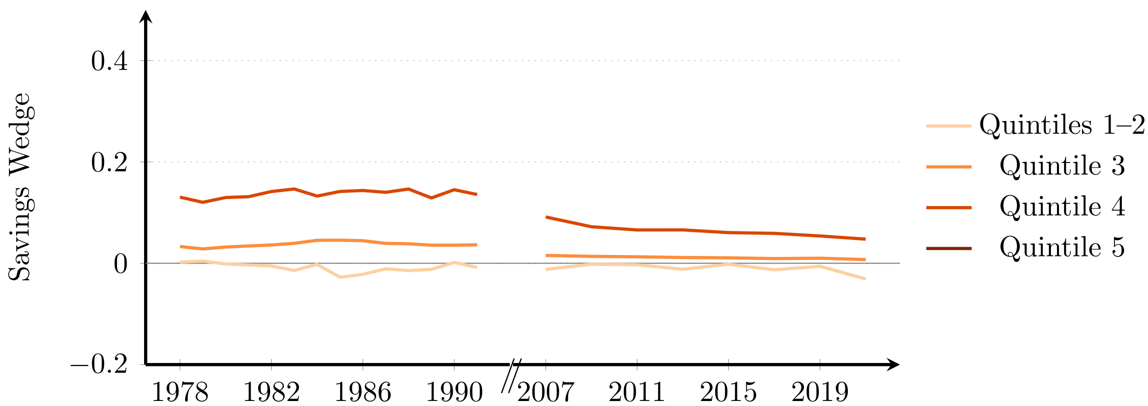


Figure 2: Capital income shares by permanent capital income quintile.



Because households' labor and capital income quintiles are highly correlated, this implies that lower income households will tend to be disproportionate wage earners while higher income households will be net capital income earners. This then implies that lower (higher) income households will contribute negatively (positively) to the idiosyncratic risk term, but positively (negatively) to the redistribution term. Therefore, whether these terms are positive or negative depends on the welfare weights assigned to each permanent income group, ν_j and $\hat{\nu}^h_j$.

Our second objective is to estimate the variances of the idiosyncratic shocks ψ and ϵ. To do this, we run separate panel regressions of log risky capital income and log labor income on individual fixed effects, year fixed effects, and an age trend. The age trend coefficient is allowed to vary by permanent income type j, where type is assigned based on the group a household belongs to at the beginning of the sample window in which they are observed. Formally, for household i of permanent type j and age h in year t, we estimate:

$$\log y_{it} = \alpha_i + \gamma_t + \beta_j \cdot h_{it} + \varepsilon_{it}, \tag{5}$$

where α_i are individual fixed effects, γ_t are year fixed effects, and β_j is a type-specific age trend. The residuals ε_{it} correspond to $\log \psi_{it}$ and $\log \epsilon_{it}$ for capital and labor income respectively. Because the log transformation is particularly sensitive to values near zero, which are likely to reflect measurement error rather than true income realizations, we restrict the regression sample to households with strictly positive income above a minimum threshold.[25] Assuming log-normality of the underlying shocks, we can recover estimates of $\text{Var}(\psi)$ and $\text{Var}(\epsilon)$ from the variance of the estimated residuals via the standard log-normal relationship $\text{Var}(x) = e^{\mu+\sigma^2/2}(e^{\sigma^2} - 1)$, where σ^2 is the variance of the log residuals.

4.2 Preliminary Estimates

We begin by estimating the wedge for a social planner who sets egalitarian welfare weights, $\nu_j = \frac{1}{\mathbb{E}[c_j]^2}$ where $\mathbb{E}[c_j]$ is the expected consumption of type-j households over all ages over all shock realizations. These weights are then normalize so that they sum to 1. We use the annualized 10-year real treasury bond rate as our baseline measure of the real rate, r.[26]

Figure 3: Savings wedge over time under egalitarian welfare weights.

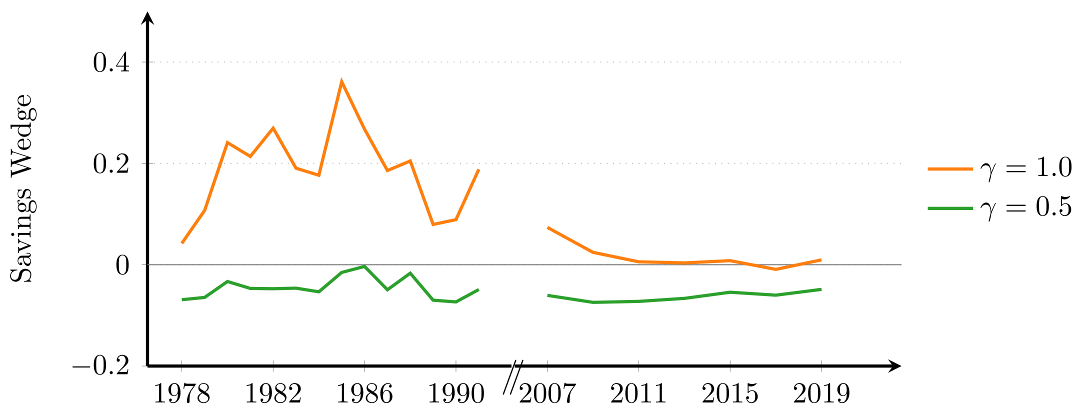


Figure 3 plots the savings wedge over time for both $\gamma = 1$ (no discounting) and $\gamma = .5$. From the figure, it is clear that shifting the Pareto weight put on future generations has a substantial impact on the savings wedge. When generations are weighted equally, the wedge suggests substantial *under-saving*, while when young generations are heavily discounted, the wedge suggests slight *over-saving*. Intuitively, these results are being driven by the redistribution effect. The young are disproportionate labor-income earners in our PSID data, benefiting from the higher wages associated with greater capital. Conversely, the old

[25]Specifically, we drop households with annual labor or capital income below \$1,000 (in 2017 USD) before taking logs.

[26]The measure of the annualized 10-year treasury bond rate come from Shiller [2026] and can be accessed at shillerdata.com.

are disproportionate capital income earners, and are negatively impacted when their existing savings earn lower returns as aggregate capital increases.

To understand the drivers of the savings wedge over time, we decompose the wedge for the $\gamma = 1$ case with egalitarian welfare weights into the risk and redistribution subcomponents. Figure 4 shows that the idiosyncratic risk term, μ_I is positive throughout our sample, implying that the contribution of the reduction in idiosyncratic capital income risk dominates the contribution of exacerbating labor income risk. This may seem surprising, as with egalitarian weights, the welfare of disproportionate labor income earners is given more weight than capital income earners. Despite this, the degree of capital income risk is substantially higher than labor income risk throughout the sample, generating a positive contribution of additional aggregate capital to idiosyncratic risk exposure.

Figure 4: Decomposition of the savings wedge into risk and redistribution.

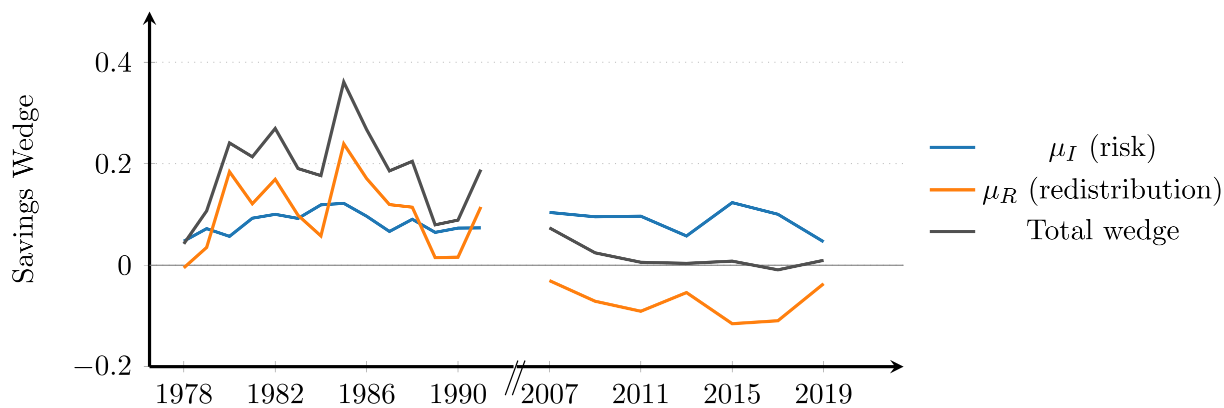
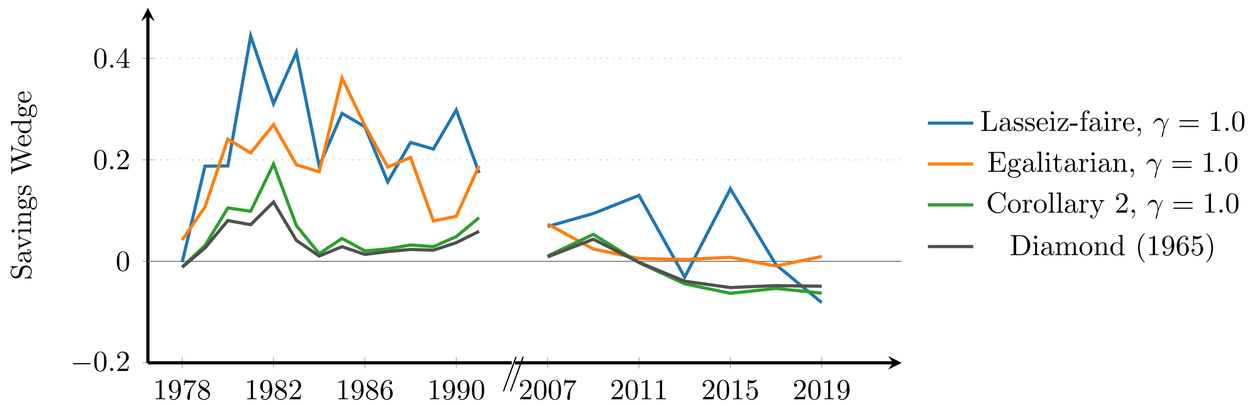


Figure 4 also shows a large positive redistribution term in the early years of the sample, that turns negative in the later years. Given that permanent income groups who are on average higher wage earners with lower capital income shares have higher welfare weights, we would expect the increase in wages associated with more capital to generate a uniformly positive redistribution term. However, the decrease in the redistribution term can be attributed to the decline in r vs. g during this period. Recall from the discussion in Section 2.2 that, as r declines relative to g, younger generations' welfare is discounted more heavily than older generations. Because the young tend to have higher ratios of labor income shares to capital income shares relative to the old, rising r relative to g implies less welfare weight on wage earners.

To put the size of our savings wedge in context, we solve for the implied savings wedge in a model without ex-ante inequality or idiosyncratic risk. The black line is our savings wedge calculated in the 2-period economy as in Diamond [1965], which corresponds to our Corollary 1. In this case, the wedge exactly reflects an r vs. g 'Modified Golden Rule' criterion, and is

Figure 5: Savings wedge vs. Golden Rule



positive – implying under-saving – whenever the annualized real 10-year bond rate is greater than the real annual growth rate. The green line is the analogous expression, but calculated in our richer model with 6-age groups in which each generations' labor and capital income shares reflect the average shares estimated in our sample.[27] This is equation 3 from Corollary 2. The orange and blue lines correspond to our savings wedge, calculated for both egalitarian welfare weights, and for 'Lasseiz-faire' welfare weights, in which $\hat{\nu}_j = 1$ for all households.[28]

From the figure, it is clear that our wedge amplifies the conclusion that the United States was under-saving in years between 1978 and 1993, and even generates a slight under-saving result in the later years when r is much closer to g. Under the Laisseiz-faire weights, all permanent income types are given the same welfare weight, dampening the redistribution term, as only the welfare benefit of inter-generational redistribution remain in the term. However, because the welfare of 'rich' high-capital income households are no longer so heavily discounted, the welfare benefits of reducing capital income risk are amplified, leading to an even larger under-saving result than in the egalitarian case.

4.2.1 Capital-skill complementarity

To be added.

[27] Recall that in Diamond [1965], the young are assumed to earn all the labor income and the old assumed to earn all the capital income.

[28] These are closer to *Lasseiz-faire or Negishi* weights in which the Pareto weights λ_j are set to eliminate the redistribution motive.

5 Conclusion

This paper derives a new criterion for evaluating over- or under-saving in economies with rich household heterogeneity. Our Savings Wedge nests the classic Golden Rule as a special case but extends it to account for how aggregate savings affects households' exposure to idiosyncratic risk and the distribution of income. A tractable second-order approximation renders the wedge measurable from standard household panel data, decomposing it into a risk channel, a redistribution channel, and a dynamic efficiency channel. Applied to PSID data, our estimates suggest that the U.S. has been substantially under-saving relative to our criterion for most of the sample period — a conclusion that the traditional Golden Rule criterion would largely miss — driven by the dominance of capital income risk over labor income risk and the higher levels of capital income inequality relative to labor income inequality.

Our findings also illuminate how structural changes in the U.S. economy over recent decades have reshaped the forces driving the savings wedge. The secular rise in markups since the 1980s has compressed the effective labor share, weakening the redistribution motive for additional saving, while rising capital income concentration has reinforced it. These forces have increasingly offset one another, gradually eroding the wedge toward the end of our sample. More broadly, our results suggest that evaluations of optimal aggregate savings based on Golden Rule criteria may substantially understate the social value of saving in economies characterized by high inequality and uninsurable idiosyncratic risk, and that the welfare consequences of savings policy depend critically on the joint distribution of labor and capital income across households.

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A Appendix

A.1 Proof of Lemma X

The individual entrepreneurs profit is:

$$\pi_{j,t} = A_t(\psi\zeta_j k_{j,t})^{\alpha}(n^d_{j,t})^{1-\alpha} - n^d_{j,t} w$$

Labor demand is:

$$n^d_{j,t} = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} \psi\zeta_j k_{j,t} A_t^{\frac{1}{\alpha}}$$

Aggregate labor is given by:

$$\ell_t = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} A_t^{\frac{1}{\alpha}} \sum_J \omega_j \zeta_j k_{j,t} \int \psi di = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} A_t^{\frac{1}{\alpha}} k_t$$

Then we can derive the equilibrium wage:

$$\ell_t^{\alpha} = (1-\alpha) A_t w_t^{-1} (k_t)^{\alpha}$$

Then wages are given by:

$$w_t = A_t(1-\alpha)\ell_t^{-\alpha} k_t^{\alpha}$$

Then an individual investors returns are given by:

$$\frac{\pi_{j,t}}{k_{j,t}} = A_t(\psi\zeta_j k_{j,t})^{\alpha}(n^d_{j,t})^{1-\alpha} - n^d_{j,t} w_t = A_t \psi\zeta_j \alpha k_t^{\alpha} \ell_t^{1-\alpha} = \psi\zeta_j r_t$$

A.2 Proof Proposition 1

Social welfare is given by:

$$\mathbb{SW} = \sum_{t=0}^{\infty} \gamma^t \sum_{j\in J} \omega_j \lambda_j \left(u\Big(\theta^y_j w_t - k_{j,t+1}\Big) + \beta \mathbb{E}\Big[u\Big(\psi\zeta_j k_{j,t+1} r_{t+1} + \epsilon\theta^o_j w_{t+1}\Big)\Big]\right).$$

The derivative of social welfare with respect to $k_{j,t+1}$ is given by the following expression:

$$\frac{d\mathbb{SW}}{dk_{j,t+1}} = \omega_j \lambda_j \Big(-u'(c^y_{j,t}) + \beta \mathbb{E}\Big[r_t \zeta_j \psi u'(c^o_{j,t+1}) \Big]\Big) + \zeta_j \sum_{\tau=0}^{\infty} \gamma^\tau \mu_{t+\tau} \frac{dk_{t+\tau}}{dk_{t+1}}$$

Where the term $\mu_{t+\tau}$ is:

$$\mu_{t+\tau} \equiv \beta \sum_j \omega_j \lambda_j \mathbb{E}\left[u'(c^o_{j,t+\tau}) \left(\psi \zeta_j k_{j,t} \frac{\partial r_{t+\tau}}{\partial k_{t+\tau}} + \epsilon \theta^o_j \frac{\partial w_{t+\tau}}{\partial k_{t+\tau}} \right) \right] + \gamma \sum_j \omega_j \lambda_j u'(c^y_{j,t+\tau}) \frac{\partial w_{t+\tau}}{\partial k_{t+\tau}} \theta^y_j$$

Using $r_{t+1} = A_{t+1} \alpha k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} = \alpha \frac{Y_{t+1}}{k_{t+1}}$ (constant along the balanced growth path) and $w_{t+1} = A_{t+1}(1-\alpha) k_{t+1}^{\alpha} n_{t+1}^{-\alpha} = (1-\alpha)\frac{Y_{t+1}}{n_t}$ (which grows at constant rate g), we obtain

$$\mu_{t+\tau} \equiv (1-\alpha) r_{t+\tau} \left\{ \beta \sum_j \omega_j \lambda_j \mathbb{E}\left[u'(c^o_{j,t+\tau}) \left(\epsilon \theta^o_j - \psi \zeta_j \frac{k_{j,t+\tau}}{k_{t+\tau}} \right) \right] + \gamma \sum_j \omega_j \lambda_j u'(c^y_{j,t+\tau}) \theta^y_j \right\}$$

Adding an intelligent zero inside the first term gives us

$$\begin{aligned} \mu_{t+\tau} \equiv (1-\alpha) r_{t+\tau} \Bigg\{ & \beta \sum_j \omega_j \lambda_j \mathbb{E}\left[u'(c^o_{j,t+\tau}) \left((\epsilon - 1)\theta^o_j - (\psi - 1)\zeta_j \frac{k_{j,t+\tau}}{k_{t+\tau}} \right) \right] \\ & + \beta \sum_j \omega_j \lambda_j \mathbb{E}\left[u'(c^o_{j,t+\tau}) \left(\theta^o_j - \zeta_j \frac{k_{j,t+\tau}}{k_{t+\tau}} \right) \right] \\ & + \gamma \sum_j \omega_j \lambda_j u'(c^y_{j,t+\tau}) \theta^y_j \Bigg\} \end{aligned}$$

Next, we note that along the balanced growth path, wages w_t grow at constant rate g, implying that the present value of resources for the current young $w_{t+1} = (1+g)w_t$, the resources available to the current old. Because preferences and rates of return remain constant along the BGP:

$$\begin{aligned} u'(c^y_{j,t}) = \beta(1+r) u'(c^o_{j,t+1}) &\implies c^o_{j,t+1} = (\beta(1+r))^{1/\eta^{RRA}} c^y_{j,t} \\ c^y_{j,t} + \frac{c^o_{j,t+1}}{1+r} = w_t \theta_j &\implies c^y_{j,t} \left(1 + \frac{(\beta(1+r))^{1/\eta^{RRA}}}{1+r} \right) = w_t \theta_j \\ &\implies c^y_{j,t} = \frac{w_t \theta_j}{\Phi(r)}, \quad \Phi(r) \equiv 1 + \frac{(\beta(1+r))^{1/\eta^{RRA}}}{1+r} \\ w_{t+1} = (1+g) w_t &\implies c^y_{j,t+1} = (1+g) c^y_{j,t} \\ &\implies u'(c^y_{j,t+1}) = \left(c^y_{j,t+1}\right)^{-\eta^{RRA}} = (1+g)^{-\eta^{RRA}} \left(c^y_{j,t}\right)^{-\eta^{RRA}} = (1+g)^{-\eta^{RRA}} u'(c^y_{j,t}) \end{aligned}$$

Therefore, we have that the final term, $\mu_{FH} = \gamma \sum_j \omega_j \lambda_j (1+g)^{-\eta^{RRA}} u'(c^y_{j,t})$. Using the households' Euler equation to sub-in for $u'(c^y_{j,t}) = (1+r)\beta u'(c^o_{j,t+1})$, we have that the final term is given by:

$$\gamma\beta \sum \lambda_j u'(c^o_{j,t+1})\theta^y_j \left(\frac{1+r}{(1+g)^{\eta^{RRA}}}\right)$$

Before we proceed, let us state the following auxiliary result.

Result 1 *Consider the expectation of the product of two functionals of random variables, i.e.,* $\mathbb{E}[f(x)g(z)]$. *Assuming* $g(\mu_z) = 0$*, to second order, we have*

$$\begin{aligned}
\mathbb{E}[f(x)g(z)] \approx \mathbb{E}\big[&f(\mu_x)g(\mu_z) + f'(\mu_x)g(\mu_z)(x-\mu_x) + f(\mu_x)g'(\mu_z)(z-\mu_z) \\
&+ \frac{1}{2}f''(\mu_x)g(\mu_z)(x-\mu_x)^2 + f'(\mu_x)g'(\mu_z)(x-\mu_x)(z-\mu_z) + \frac{1}{2}f(\mu_x)g''(\mu_z)(z-\mu_z)^2\big] \\
&= f'(\mu_x)g'(\mu_z)Cov(x,z) \ ,
\end{aligned}$$

where the last line follows from $g(z) = z$ *with* $g'(\mu_z) = 1$, $g''(\mu_z) = 0$ *and* $g(\mu_z) = 0$.

First Term: Using the second period household budget constraint, i.e., $c^o_{j,t+1} = \psi\zeta_j r_{t+1}k_{j,t+1} + \epsilon\theta^o_j w_{t+1}$, with $\mathbb{E}\left[c^o_{j,t+1}\right] = \zeta_j r_{t+1}k_{j,t+1} + \theta^o_j w_{t+1}$, $f(c^o_{j,t+1}) \equiv u'(c^o_{j,t+1})$, and $z \equiv (\epsilon-1)\theta^o_j - (\psi-1)\zeta_j \frac{k_{j,t+1}}{k_{t+1}}$ we obtain:

$$\begin{aligned}
\mathbb{E}\left[f(c^o_{j,t+1})z\right] &\approx u''(\mathbb{E}\left[c^o_{j,t+1}\right])\text{Cov}\left(\psi\zeta_j r_{t+1}k_{j,t+1} + \epsilon\theta^o_j w_{t+1}, (\epsilon-1)\theta^o_j - (\psi-1)\zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right) \\
&= u''(\mathbb{E}\left[c^o_{j,t+1}\right])\text{Cov}\left(\psi\zeta_j r_{t+1}k_{j,t+1} + \epsilon\theta^o_j w_{t+1}, \epsilon\theta^o_j - \psi\zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right) \ ,
\end{aligned}$$

where we can write

$$\begin{aligned}
&\text{Cov}\left(\psi\zeta_j r_{t+1}k_{j,t+1} + \epsilon\theta^o_j w_{t+1}, \epsilon\theta^o_j - \psi\zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right) \\
&= \left[(\theta^o_j)^2 w_{t+1}\right]\text{Var}(\epsilon) - \left[(\zeta_j k_{j,t+1})^2\frac{r_{t+1}}{k_{t+1}}\right]\text{Var}(\psi) + \left[\theta^o_j\zeta_j r_{t+1}k_{j,t+1} - \theta^o_j w_{t+1}\zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right]\text{Cov}(\epsilon,\psi) \\
&= Y_{t+1}\left\{(1-\alpha)(\theta^o_j)^2\text{Var}(\epsilon) - \alpha\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}}\right)^2\text{Var}(\psi) + (2\alpha-1)\theta^o_j\frac{\zeta_j k_{j,t+1}}{k_{t+1}}\text{Cov}(\epsilon,\psi)\right\}
\end{aligned}$$

Second Term: To second order, we obtain

$$\mathbb{E}\left[u'(c^o_{j,t+1})\left(\theta^o_j - \zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right)\right] \approx \left(\theta^o_j - \zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right)\left\{u'(\mathbb{E}\left[c^o_{j,t+1}\right]) + \frac{1}{2}u'''(\mathbb{E}\left[c^o_{j,t+1}\right])\mathrm{Var}(c^o_{j,t+1})\right\}$$

which can be simplified to

$$\begin{aligned}\mathbb{E}\left[u'(c^o_{j,t+1})\left(\theta^o_j - \zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right)\right] &\approx u'(\mathbb{E}\left[c^o_{j,t+1}\right])\left(\theta^o_j - \zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right)\left\{1 + \frac{1}{2}\frac{u'''(\mathbb{E}\left[c^o_{j,t+1}\right])}{u'(\mathbb{E}\left[c^o_{j,t+1}\right])}\mathrm{Var}(c^o_{j,t+1})\right\}\\ &= u'(\mathbb{E}\left[c^o_{j,t+1}\right])\left(\theta^o_j - \zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right)\left\{1 + \frac{1}{2}\eta^{RRA}\eta^{RPR}\frac{\mathrm{Var}(c^o_{j,t+1})}{\mathbb{E}\left[c^o_{j,t+1}\right]^2}\right\},\end{aligned}$$

where we have defined the coefficients of relative risk aversion (η^{RRA}) and relative prudence (η^{RPR}) as follows (see Kimball 1990, ECMA):

$$\begin{aligned}\eta^{RRA} &\equiv -\mathbb{E}\left[c^o_{j,t+1}\right]\frac{u''(\mathbb{E}\left[c^o_{j,t+1}\right])}{u'(\mathbb{E}\left[c^o_{j,t+1}\right])} > 0\,,\\ \eta^{RPR} &\equiv -\mathbb{E}\left[c^o_{j,t+1}\right]\frac{u'''(\mathbb{E}\left[c^o_{j,t+1}\right])}{u''(\mathbb{E}\left[c^o_{j,t+1}\right])} > 0\,.\end{aligned}$$

Moreover, we can compute the variance of consumption according to

$$\begin{aligned}\mathrm{Var}(c^o_{j,t+1}) &= \mathrm{Var}(\psi\zeta_j r_{t+1}k_{j,t+1} + \epsilon\theta^o_j w_{t+1})\\ &= Y^2_{t+1}\left\{\alpha^2\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}}\right)^2\mathrm{Var}(\psi) + (1-\alpha)^2(\theta^o_j)^2\mathrm{Var}(\epsilon) + 2\alpha(1-\alpha)\theta^o_j\frac{\zeta_j k_{j,t+1}}{k_{t+1}}\mathrm{Cov}(\epsilon,\psi)\right\}\end{aligned}$$

A.2.1 Measurable Savings Wedge

Substituting in all approximated terms, the *measurable* savings wedge is to second order given by:

$$\mu_{t+1} \approx (1-\alpha)r_{t+1}\beta\Big[\mu_{PE} + \mu_{RE} + \mu_{FH}\Big]\,,$$

where the sub-components are given by:

$$\mu_{PE} \approx \sum_j \omega_j \lambda_j \eta^{RRA} u'(\mathbb{E}\left[c^o_{j,t+1}\right]) \frac{Y_{t+1}}{\mathbb{E}\left[c^o_{j,t+1}\right]}$$
$$\times \left\{ \alpha \left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}} \right)^2 \mathrm{Var}(\psi) - (1-\alpha)(\theta^o_j)^2 \mathrm{Var}(\epsilon) + (1-2\alpha)\theta^o_j \frac{\zeta_j k_{j,t+1}}{k_{t+1}} \mathrm{Cov}(\epsilon, \psi) \right\}$$

$$\mu_{RE} \approx \sum_j \omega_j \lambda_j u'(\mathbb{E}\left[c^o_{j,t+1}\right]) \left(\theta^o_j - \zeta_j \frac{k_{j,t+1}}{k_{t+1}} \right)$$
$$\times \left\{ 1 + \frac{1}{2} \eta^{RRA} \eta^{RPR} \left(\frac{Y_{t+1}}{\mathbb{E}\left[c^o_{j,t+1}\right]} \right)^2 \left[\alpha^2 \left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}} \right)^2 \mathrm{Var}(\psi) + (1-\alpha)^2 (\theta^o_j)^2 \mathrm{Var}(\epsilon) + 2\alpha(1-\alpha)\theta^o_j \frac{\zeta_j k_{j,t+1}}{k_{t+1}} \mathrm{Cov}(\epsilon, \psi) \right] \right\}$$

$$\mu_{FH} = \sum_j \lambda_j u'(\mathbb{E}[c^o_{j,t+1}]) \theta^y_j \left(\frac{1+r}{(1+g)^{\eta^{RRA}}} \right) \gamma$$

A.2.2 Proof of Corollary 1.

Assume as stated in the Proposition that second period labor is 0, and that all ex-ante heterogeneity and risk is eliminated. By construction, then $\theta^y = 1$. Then the expression from Proposition 1 becomes:

$$\mu_{t+\tau} \propto r_{t+\tau} \left(-u'(c^o_t)\beta + \frac{\gamma}{(1+g)^{\eta^{RRA}}} u'(c^y_t) \right)$$

Plugging in the Euler equation, evaluating the above at $\gamma = 1$ and log utility gives the result.:

$$\mu_{t+\tau} \propto r_{t+\tau} u'(c^y_t) \left(\frac{-1}{1+r_{t+\tau}} + \frac{\gamma}{1+g} \right)$$

A.3 Interaction of the risk and redistribution effects.

Suppose that the $\mathrm{Cov}(\psi, \epsilon)$=0. Then the second-order approximation of the savings wedge around the zero-risk equilibrium is given by the following expression.

$$\mu_{t+1} \approx (1-\alpha) r_{t+1} \left[\mu_I + \mu_R + \mu_D \right],$$

where the sub-components are given by:

$$\mu_I = \beta\eta^{RRA}\sum_j \omega_j\nu_j \frac{Y_{t+1}}{\mathbb{E}\left[c^o_{j,t+1}\right]} \times \left(\alpha\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}}\right)^2 \mathrm{Var}(\psi) - (1-\alpha)(\theta^o_j)^2\mathrm{Var}(\epsilon)\right)$$

$$\mu_R = \beta\sum_j \omega_j\nu_j\left(\theta^o_j - \zeta_j\frac{k_{j,t+1}}{k_{t+1}}\right)\left(1+\frac{1}{2}\eta^{RRA}\eta^{RPR}\frac{\mathrm{Var}(c^o_{j,t+1})}{\mathbb{E}\left[c^o_{j,t+1}\right]^2}\right)$$

$$\mu_D = \gamma\sum_j \omega_j\nu^y_j\theta^y_j$$

Definition. Define the two auxiliary welfare weights:

$$\tilde{\nu}_j \equiv \nu_j\frac{Y_{t+1}}{\omega_j\mathbb{E}[c^o_{j,t+1}]}$$

$$\hat{\nu}_j \equiv \nu_j\left(1+\frac{1}{2}\eta^{RRA}\eta^{RPR}\frac{\mathrm{Var}(c^o_{j,t+1})}{\mathbb{E}\left[c^o_{j,t+1}\right]^2}\right)$$

Note that μ_I can be written in the following way:

$$\mu_I = \beta\eta^{RRA}\ \sum_j\tilde{\nu}_j\left(\alpha\left(\frac{\omega_j\zeta_j k_{j,t+1}}{k_{t+1}}\right)^2 Var(\psi) - (1-\alpha)(\omega_j\theta^o_j)^2 Var(\epsilon)\right) = \beta\eta^{RRA}\ \sum_j\tilde{\nu}_j\Omega_j$$

where Ω_j is defined as:

$$\Omega_j \equiv \left(\alpha\left(\frac{\omega_j\zeta_j k_{j,t+1}}{k_{t+1}}\right)^2 Var(\psi) - (1-\alpha)(\omega_j\theta^o_j)^2 Var(\epsilon)\right)$$

Similarly, μ_R can similarly be written as:

$$\mu_R = \beta\sum_j\hat{\nu}_j\omega_j\left(\theta^o_j - \frac{\zeta_j k_{j,t+1}}{k_{t+1}}\right)$$

A.3.1 Derivative of μ_I and μ_R to changes in the capital income distribution

Consider a set of infinitesimal changes to the distribution of capital income, $\zeta_j k_j + \Delta_j$ where $\sum_j \Delta_j = 0$, leaving off time subscripts for simplicity. We will characterize the distribution of changes, $\{\Delta_j\}_{j\in J}$ below.

For notational simplicity, we denote $c_j \equiv \mathbb{E}[c^o_{j,t+1}]$

Solve for $d\mu_I$. Consider small changes Δ_j to capital holdings $\zeta_j k_j$ such that $\sum_{j=1}^{J} \omega_j \Delta_j = 0$ (preserving total capital K). The first-order change in μ_I is:

$$\Delta\mu_I = \sum_{j=1}^{J} \Delta_j \cdot \frac{\partial \mu_I}{\partial(\zeta_j k_j)} = \sum_j \left(\Omega_j \Delta\tilde{v}_j + \tilde{v}_j \Delta\Omega_j\right)$$

Since $\tilde{\nu}_j = \lambda_j \frac{u'(c_j)Y_{t+1}}{\omega_j c_j}$, we have:

$$\begin{aligned}
\Delta\tilde{\nu}_j &= \frac{\lambda_j Y_{t+1}}{\omega_j}\left[\frac{u''(c_j)\Delta c_j \cdot c_j - u'(c_j)\Delta c_j}{c_j^2}\right] \\
&= \frac{\lambda_j Y_{t+1}\Delta c_j}{\omega_j c_j}\left[u''(c_j) - \frac{u'(c_j)}{c_j}\right] \\
&= \frac{\lambda_j Y_{t+1} r \Delta_j}{\omega_j c_j}\left[u''(c_j) - \frac{u'(c_j)}{c_j}\right]
\end{aligned}$$

The change in Ω_j is:

$$\Delta\Omega_j = \alpha Var(\psi)\frac{\omega_j^2}{K^2} 2\zeta_j k_j$$

The change in μ_I can be decomposed into a re-weighting effect and a concentration effect:

$$\Delta\mu_I = \Delta\mu_I^{\text{reweight}} + \Delta\mu_I^{\text{concentr}}$$

Where the reweighting effect is given by:

$$\Delta\mu_I^{\text{reweight}} = \sum_{j=1}^{J} \Delta_j \cdot \frac{\lambda_j Y_{t+1} r \Omega_j}{\omega_j c_j}\left(u''(c_j) - \frac{u'(c_j)}{c_j}\right)$$

And the concentration effect is (assuming uniform ω_j):

$$\Delta\mu_I^{\text{concentr}} = \sum_{j=1}^{J} \Delta_j \cdot \frac{2\alpha Var(\psi)\tilde{\nu}_j \omega_j^2 \zeta_j k_j}{K^2} = 2\alpha Var(\psi)\frac{\omega}{K}\sum_{j=1}^{J} \Delta_j \frac{\tilde{\nu}_j \omega_j \zeta_j k_j}{K}$$

Note that when relative inequality is 0 but absolute inequality is not, we have that:

$$\begin{aligned}
Var\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}}\frac{1}{\theta_j}\right) &= 0 \text{ and} \\
Var\left(c^o_{j,t+1}\right) &> 0
\end{aligned}$$

This implies that $\frac{\zeta_j k_j}{k} = \theta_j$ and therefore that $\frac{\Omega_j}{c_j^2}$ is the same for all household types (intuitively, households capital income share = their labor share = their consumption share). Therefore we can write the the reweighting term as:

$$\Delta\mu_I^{\text{reweight}} = \frac{\Omega}{c^2} \sum_{j=1}^{J} \Delta_j \cdot \frac{ru'(c_j)}{\omega_j} \left(-\eta^{RRA} - 1\right)$$

Because $\Delta_j > 0$ for those with high $\zeta_j k_j$ and therefore high c_j and low $u'(c_j)$, the reweighting term is unambiguously negative when $\alpha Var(\psi) < (1-\alpha)Var(\epsilon)$. Intuitively, when labor income risk is more severe than capital income risk, the idiosyncratic risk term is negative, as higher wages and lower average returns amplify risk exposure. When households with a high welfare weight are given an even smaller share of capital income, this amplifies their net exposure, making the impact on welfare through idiosyncratic risk even more severe. If capital income risk is more severe, then further concentrating risk among high income households and removing exposure from low income households improves welfare and increases μ_I.

The sign of the concentration term in this case is unambiguously negative, as $\Delta\mu_I^{concentr}$ is given by:

$$\Delta\mu_I^{concentr} \propto \frac{Y_{t+1}}{\mathbb{E}[c^o_{j,t+1}]} \frac{\omega_j \zeta_j k_{j,t+1}}{k_{t+1}} \sum_{j=1}^{J} \Delta_j u'(\mathbb{E}[c^o_{j,t+1}]) < 0$$

Where here we again use the fact that in the case with no relative inequality, all households have the same ratio between their capital income share and their consumption share.

Let $\Omega = \alpha Var(\psi) - (1-\alpha)Var(\epsilon)$. Then the total change in μ_I can be written as:

$$\Delta\mu_I = \sum_{j=1}^{J} \Delta_j \omega_j u'(\theta_j Y) \left[-\frac{r\Omega(\eta^{RRA}+1)}{Y} + \frac{2\alpha \text{Var}(\psi)}{K} \right]$$

Whenever Ω is negative (labor risk dominates), this change is unambiguously negative. Whenever the following condition is met:

$$\left(\frac{2}{K} - \frac{(1+\eta)r}{Y}\right) = K\left(2 - (1+\eta)\alpha\right) \geq 0$$

Then the term is unambiguously negative as well. For $\alpha \approx \frac{1}{3}$ and $\eta \leq 2$ this condition is satisfied.

Solve for $d\mu_R$. We can write μ_R as:

$$\mu_R = \beta\Bigg(\text{mean}(\hat{\nu}_j)\text{mean}\Bigg(\omega_j\theta_j^o - \frac{\omega_j\zeta_j k_{j,t+1}}{k_{t+1}}\Bigg) + Cov\Bigg(\hat{\nu}_j, \Bigg(\omega_j\theta_j^o - \frac{\omega_j\zeta_j k_{j,t+1}}{k_{t+1}}\Bigg)\Bigg)\Bigg)$$

$$= \beta Cov\Bigg(\hat{\nu}_j, \Bigg(\omega_j\theta_j^o - \frac{\omega_j\zeta_j k_{j,t+1}}{k_{t+1}}\Bigg)\Bigg)$$

Note that here, we're using the fact that the average difference between the share of labor income and the share of capital income is by construction equal to 0. Then we need to calculate the difference in μ_I and the difference in μ_R after a given change in the capital income distribution.

This term is easier to sign. If we can establish conditions under which $\hat{\nu}_j$ is higher for already low income households, then giving more capital income to high income households straightforwardly increases the above Covariance.

Expanding out the $\hat{\nu}_j$ term (again assuming 0 Covariance):

$$\hat{\nu}_j = \nu_j\Bigg(1 + \frac{1}{2}\eta^{RRA}\eta^{RPR}\left(\frac{Y_{t+1}}{\mathbb{E}\left[c^o_{j,t+1}\right]}\right)^2\left[\alpha^2\left(\frac{\zeta_j k_{j,t+1}}{k_{t+1}}\right)^2\text{Var}(\psi) + (1-\alpha)^2(\theta_j^o)^2\text{Var}(\epsilon)\right]\Bigg)$$

If as in the text, we assume no relative inequality, then:

$$\frac{\zeta_j k_{j,t+1}}{k_{t+1}} = \theta_j \quad \forall\, j \in J$$

In this case, $\hat{\nu}_j \propto \nu_j$ and high-income households have unambiguously higher $\hat{\nu}_j$. In this case, the change in the capital income distribution $\{\Delta_j\}_{j\in J}$ assumed above would unambiguously increase μ_R by increasing $Cov\Big(\hat{\nu}_j, \Big(\omega_j\theta_j^o - \frac{\omega_j\zeta_j k_{j,t+1}}{k_{t+1}}\Big)\Big)$.

A.4 General Model Results.

A.4.1 Proof of Lemma 2

The final good firm's problem is a standard problem whose solution implies that firm n's demand and the aggregate price level are given by the following expressions.

$$y_{nt} = \left(\frac{p_{nt}}{P_t}\right)^{-\rho} y_t \qquad P_t = \left(\int p_{nt}^{1-\rho} di\right)^{\frac{1}{1-\rho}} \tag{6}$$

Private entrepreneurs. Private businesses earn the private stochastic return $r_{j,t}$, which we derive below. Entrepreneurs operate the following production function with productivity shock, ψ. We define the nominal wage, $W_t = w_t P_t$.

$$y_{j,t} = (\psi\nu_j)^{\alpha}(k_{j,t})^{\alpha}(\ell^d_{j,t})^{1-\alpha}$$

For a given capital stock, $k_{j,t}$, entrepreneurs profit in period t is given by:

$$\pi_{j,t} = p_{j,t}(y_{j,t})y_{j,t} - W_t\ell^d_{j,t} + P_t k_{j,t}(1-\delta)$$

Solving for labor demand as a function of ψ, the *nominal* wage W_t, and $k_{j,t}$:

$$\left(\frac{\partial p_{j,t}}{\partial y_{j,t}}y_{j,t} + p_{j,t}\right)\frac{\partial y_{j,t}}{\partial \ell^d_{j,t}} - W_t = 0$$

Plugging in the production function and the firm's demand function and dividing by P_t gives:

$$\left(\mathcal{M}\right)\frac{p_{j,t}}{P_t}\psi^{\alpha}\nu_j^{\alpha}(1-\alpha)(k_{j,t})^{\alpha}(\ell^d_{j,t})^{-\alpha} = w_t$$
$$\left(\left(\mathcal{M}\right)(1-\alpha)w_t^{-1}\right)^{\frac{1}{\alpha}}\psi\nu_j k_{j,t} = \ell^d_{j,t}$$

Where we assume that all firms set the same price in equilibrium (verify this!)[29]. Using the labor market clearing condition to solve for the equilibrium wage:

$$\ell_t = \sum_J \omega_j \ell^d_{j,t} + \ell^c_t$$
$$\rightarrow w_t = \mu(1-\alpha)\left(k^c_t + \sum_J \omega_j\nu_j k_{j,t}\right)^{\alpha}\ell_t^{-\alpha}$$

Subbing the expression for labor demand in, we can write (real) returns as:

$$r_{j,t} = \frac{\pi_{j,t}}{k_{j,t}} = \psi\nu_j\left(\left(\frac{\frac{(\rho-1)}{\rho}(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}} - w_t\left(\frac{\mathcal{M}(1-\alpha)}{w_t}\right)^{\frac{1}{\alpha}}\right) + 1 - \delta$$
$$r_{j,t} = \psi\nu_j\left(\frac{1}{k_t^{\alpha}\ell_t^{-\alpha}}\right)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)k_t^{\alpha}\ell^{1-\alpha}\left(\frac{1}{k_t^{\alpha}\ell_t^{-\alpha}}\right)^{\frac{1}{\alpha}} + 1 - \delta$$
$$r_{j,t} = \alpha\psi\nu_j k_t^{\alpha-1}\ell_t^{1-\alpha} + 1 - \delta$$

[29] Or just assume that prices have to be set in expectation

Here, we define aggregate capital, k_t as in the text:

$$k_t = \left(\sum \omega_j \int_I \nu_j k^e_{ijt} + k^c_{ijt} \right)$$

Corporate sector. A financial intermediary pools together equity from a measure of corporate firms and offers the return r^c_t. The rate of return on capital invested in the corporate sector, inclusive of profits, is:

$$r^c_t = \alpha k_t^{\alpha-1} \ell_t^{1-\alpha} + 1 - \delta$$

Deriving the bond interest rate. We can use (i) the bond market clearing condition, (ii) households' bond Euler equations, (iii) and the no-arbitrage condition between capital and bonds to derive an implicit function of the bond market.

A.5 Deriving the general savings wedge.

Now the social welfare function is given by:

$$SW = \sum_{t=0}^{\infty} \gamma^t \sum_{j \in J} \omega_j \lambda_j \left(\sum_{h=0}^{A} \beta^h \mathbb{E}[u(c^h_{j,t+h})] \right)$$

subject to:

$$\begin{aligned} c^h_{j,t+h} = (1-\tau_{\ell,t+h})\theta^h_j \epsilon w_{t+h} - b^h_{j,t+h+1} - k^{ch}_{j,t+h+1} - k^h_{j,t+h+1} + (1-\tau_{k,t+h})((1+r^b_{t+h})b^h_{j,t+h} + T_t \\ +(1+r^c_{t+h})k^{ch}_{j,t+h} + (1+r_{j,t+h})k^h_{j,t+h}) \\ r^c_t = \alpha k_t^{\alpha-1}\ell_t^{1-\alpha} + 1 - \delta \qquad r_{j,t} = \alpha\psi\nu_j k_t^{\alpha-1}\ell_t^{1-\alpha} + 1 - \delta \qquad w_t = \mu(1-\alpha)k_t^{\alpha}\ell_t^{-\alpha} \end{aligned}$$

We assume that $B_t = B$ and that a uniform lump-sum tax, G_t absorbs any changes in revenue.

The total marginal value of additional saving from type-j households born in period t, $\frac{dSW}{dk_t}$ is equal to the sum of the *direct* effect on households' private utility plus an *indirect* *effect* or *wedge*, μ_{t+1} through changes in factor prices.

$$\frac{dSW}{dk^h_{j,t}} = \omega_j\zeta_j\Bigg(\sum_j\sum_h \gamma^{t-h+1}\omega_j\lambda_j\beta^{h-1}\Big(-u'(c^{h-1}_{j,t-1}) + \beta(1-\tau_{kt})\mathbb{E}\Big[(1+r_t\zeta_j\psi)u'(c^h_{j,t})\Big]$$
$$-u'(c^{h-1}_{j,t-1})\frac{db^h_{j,t}}{dk^h_{j,t}} + \beta(1-\tau_{kt})(1+r_t)\mathbb{E}\Big[u'(c^h_{j,t})\Big]\frac{db^h_{j,t}}{dk^h_{j,t}}$$
$$-u'(c^{h-1}_{j,t-1})\frac{dk^{ch}_{j,t}}{dk^h_{j,t}} + \beta(1-\tau_{kt})(1+r^c_t)\mathbb{E}\Big[u'(c^h_{j,t})\Big]\frac{dk^{ch}_{j,t}}{dk^h_{j,t}}\Big) + \mu_t\Bigg)$$

We evaluate this expression at the lasseiz-faire equilibrium, implying that $\frac{dSW}{dk^h_{j,t}} = \omega_j\zeta_j\mu_t$. Similarly, $\frac{dSW}{dk^{hc}_{j,t}} = \omega_j\mu_t$. Then if dk_t can be written as:

$$dk_t = \sum_j\sum_h \omega_j\Big(\zeta_j dk^h_{j,t} + dk^{hc}_{j,t}\Big).$$

Then we have that $\frac{dSW}{dk_t} = \mu_t$. Where μ_t is given by:

$$\mu_t = \Bigg(\sum_{h=1}^{A}\sum_j\Big(\gamma^{t-h}\beta^h\lambda_j\omega_j\mathbb{E}\Big[u'(c^h_{j,t})\Big((1-\tau_{\ell,t})\theta^h_j\epsilon\frac{dw_t}{dk_t} +$$
$$(1-\tau_{kt})\Big(b^h_{j,t}\frac{dr^b_t}{dk_t} + k^{ch}_{j,t}\frac{dr^c_t}{dk_{j,t}} + \zeta_j k^h_{j,t}\psi\frac{dr_t}{dk_t}\Big) + \Big)\Big] + \gamma^t u'(c^0_{j,t})(1-\tau_{\ell t})\sum_j\theta^0_j\frac{dw_t}{dk_t}\Bigg)$$

Recall that $r_t = A_t\alpha k^{\alpha-1}_t\ell^{1-\alpha}_t$ and $w_t = A_t\mathcal{M}(1-\alpha)k^{\alpha}_t\ell^{-\alpha}_t$. Therefore, $\frac{dw_t}{dk_t} = A_t\mathcal{M}(1-\alpha)\ell^{1-\alpha}_t k^{\alpha-1}_t\alpha\ell^{-1}_t = \mathcal{M}r_t(1-\alpha)\ell^{-1}_t$ and $\frac{dr_t}{dk_t} = \alpha(\alpha-1)r_t k^{-1}_t$. Following the same procedure as before, we can decompose μ_t into risk and redistribution terms by adding an intelligent zero.

$$\mu_t = (1-\alpha)r_{t+1}\Bigg(\sum_j\omega_j\lambda_j\sum_{h=1}^{A}\gamma^{t-h}\beta^h\mathbb{E}\Big[u'(c^h_{j,t})\Big((\epsilon-1)\theta^h_j\mathcal{M}(1-\tau_{\ell t}) - (1-\tau_k)(\psi-1)\zeta_j\frac{k_{j,t}}{k_t}\Big)\Big]$$
$$+\sum_j\omega_j\lambda_j\sum_{h=1}^{A}\gamma^{t-h}\beta^h\mathbb{E}\Big[u'(c^h_{j,t})\Big]\Big(\theta^h_j\mathcal{M}(1-\tau_{\ell t}) - \Big(\zeta_j\frac{k^h_{j,t}}{k_t} - \frac{k^{ch}_{j,t}}{k_t} - \frac{b^h_{j,t}}{k_t}\Big)(1-\tau_{kt})\Big)$$
$$+\sum_j\omega_j\lambda_j u'(c^0_{j,t})\gamma^t(1-\tau_{\ell t})\mathcal{M}\theta^0_j\Bigg)$$

Households' Euler equation for the risk-free asset is:

$$u'(c_{j,t}^h) = \beta(1+r)\mathbb{E}[u'(c_{j,t+1}^{h+1})].$$

Due to economic growth, $\mathbb{E}[u'(c_{j,t+1}^{h+1})] = (1+g)^{-\eta^{RRA}}\mathbb{E}[u'(c_{j,t}^{h+1})]$

Subbing this in above, and applying the law of iterated expectations:

$$\mu_t \propto r_{t+1}\beta^H \Bigg(\sum_j \omega_j \lambda_j \sum_{h=1}^{A} \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}}\right)^h \mathbb{E}\Bigg[u'(c_{j,t}^H)\Bigg((\epsilon-1)\theta_j^h \mathcal{M}(1-\tau_{\ell t}) - (1-\tau_k)(\psi-1)\zeta_j \frac{k_{j,t}^h}{k_t}\Bigg)\Bigg]$$
$$+\sum_j \omega_j \lambda_j \sum_{h=1}^{A} \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}}\right)^h \mathbb{E}\Big[u'(c_{j,t}^H)\Big]\Bigg(\theta_j^h \mathcal{M}(1-\tau_{\ell t}) - \Bigg(\zeta_j \frac{k_{j,t}^h}{k_t} - \frac{k_{j,t}^{ch}}{k_t} - \frac{b_{j,t}^h}{k_t}\Bigg)(1-\tau_{kt})\Bigg)$$
$$+\sum_j \omega_j \lambda_j \mathbb{E}[u'(c_{j,t}^H)]\left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}}\right)^H (1-\tau_{\ell t})\theta_j^0 \mathcal{M}\Bigg)$$

Recall that if we define $z \equiv \Big((\epsilon-1)\theta_j^h \mathcal{M}(1-\tau_{\ell t}) - (1-\tau_k)(\psi-1)\zeta_j \frac{k_{j,t}}{k_t}\Big)$ then we have that:

$$\mathbb{E}\Big[u'(c_{j,t}^h)\times z\Big] \approx u''(\mathbb{E}[c_{j,t}^h])\text{Cov}\Big(c_{j,t}^h, z\Big)$$
$$= \Big(\theta_j^h(1-\tau_{\ell t})\Big)^2 \text{Var}(\epsilon)\mathcal{M}w_t\ell_t - \left(\frac{\rho_j k_{j,t}^h}{k_t}(1-\tau_{kt})\right)^2 \text{Var}(\psi) r_t k_t -$$
$$\text{Cov}(\epsilon,\psi)\theta_j^h \frac{\zeta_j k_{j,t}^h}{k_t}(1-\tau_{\ell t})(1-\tau_{kt)}\Big(\mathcal{M}r_t k_t - w_t \ell_t\Big)$$

Taking a second order approximation of $\mathbb{E}[u'(c_{j,t}^h)]$ around the zero-risk steady state gives:

$$\mathbb{E}[u'(c_{j,t}^h)] \approx u'(\mathbb{E}[c_{j,t}^h])\left(1 + \frac{1}{2}\eta^{RRA}\eta^{RPR}\frac{Var(c_{j,t}^h)}{\mathbb{E}[c_{j,t}^h]^2}\right)$$

Dividing through by $y_t = k_t^\alpha \ell_t^{1-\alpha}$ and noting that $u''(\mathbb{E}[c_{j,t}^h]) = -\eta^{RRA}\frac{u'(\mathbb{E}[c_{j,t}^h])}{\mathbb{E}[c_{j,t}^h]}$

$$\mu_t \approx (1-\alpha) r_{t+1} \beta^H \Bigg(\sum_j \omega_j \lambda_j \sum_{h=1}^{A} \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}} \right)^h u'(\mathbb{E}[c_{j,t}^H]) \frac{y_t}{\mathbb{E}[c_{j,t}^H]} \Bigg(- \Big(\theta_{j,t}^h (1-\tau_{\ell t}) \mathcal{M} \Big)^2 (1-\alpha) \text{Var}(\epsilon)$$

$$+ \Big(\frac{\zeta_j k_{j,t}^h}{k_t} (1-\tau_{kt}) \Big)^2 \alpha \text{Var}(\psi) - \Big(\mathcal{M}\alpha + \alpha - 1 \Big) (1-\tau_{\ell t})(1-\tau_{kt}) \theta_j^h \frac{\zeta_j k_{h,t}^h}{k_t} \text{Cov}(\epsilon, \psi) \Bigg)$$

$$+ \sum_j \omega_j \lambda_j \sum_{h=0}^{A} \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}} \right)^h \Bigg(\Big(\theta_j^h \mathcal{M} (1-\tau_{\ell t}) - \Big(\zeta_j \frac{k_{j,t}^h}{k_t} - \frac{k_{j,t}^{ch}}{k_t} - \frac{b_{j,t}^h}{k_t} \Big) (1-\tau_{kt}) \Big) \times$$

$$u'(\mathbb{E}[c_{j,t}^H]) \Big(1 + \frac{1}{2} \eta^{RRA} \eta^{RPR} \frac{Var(c_{j,t}^H)}{\mathbb{E}[c_{j,t}^H]^2} \Big)$$

Finally, defining $\nu_{j,t} \equiv \lambda_j u'(\mathbb{E}[c_{j,t}^H])$ and $\hat{\nu}_{j,t} \equiv \nu_{j,t} \Big(1 + \frac{1}{2} \eta^{RRA} \eta^{RPR} \frac{Var(c_{j,t}^H)}{\mathbb{E}[c_{j,t}^H]^2} \Big)$, we can re-write the above.

$$\mu_t \approx (1-\alpha) r_{t+1} \Big(\mu_{It} + \mu_{Rt} \Big)$$

where

$$\mu_{It} = \sum_j \sum_h \nu_{j,t} \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}} \right)^h \frac{y_t}{\omega_j \mathbb{E}[c_{j,t}^H]} \Bigg(\alpha \text{Var}(\psi) \Big(\frac{\omega_j \zeta_j k_{j,t}^h (1-\tau_{kt})}{k_t} \Big)^2$$

$$- (1-\alpha) \text{Var}(\epsilon) \Big(\omega_j \theta_j^h (1-\tau_{\ell t}) \mathcal{M} \Big)^2 - \Big(\mathcal{M}\alpha + \alpha - 1 \Big) (1-\tau_{\ell t})(1-\tau_{kt}) \omega_j \theta_j^h \frac{\omega_j \zeta_j k_{h,t}^h}{k_t} \text{Cov}(\epsilon, \psi) \Bigg)$$

$$\mu_{Rt} = \sum_j \sum_h \hat{\nu}_{j,t} \left(\frac{\gamma(1+r)}{(1+g)^{\eta^{RRA}}} \right)^h \Bigg(\omega_j \theta_j (1-\tau_{\ell t}) \mathcal{M} - \Big(\frac{\zeta_j k_{j,t}^h}{k_t} + \frac{k_{j,t}^{hc}}{k_t} + \frac{b_t}{k_t} \Big) (1-\tau_{kt}) \Bigg)$$

A.5.1 Capital-skill complementarity

The production function is

$$Y = [\lambda(\ell^s)^\sigma + (1-\lambda) Z^\sigma]^{1/\sigma}, \qquad Z = (\mu K^\rho + (1-\mu)(\ell^c)^\rho)^{1/\rho}$$

with factor prices $r = \partial Y / \partial K$, $w^c = \partial Y / \partial \ell^c$, $w^s = \partial Y / \partial \ell^s$, and define

$$q \equiv \frac{\partial Y}{\partial Z} = (1-\lambda) Z^{\sigma-1} Y^{1-\sigma}, \qquad s_K^Z \equiv \frac{rK}{rK + w^c \ell^c}, \qquad s_Z^Y \equiv \frac{rK + w^c \ell^c}{Y}, \qquad s_{\ell^s}^Y \equiv \frac{w^s \ell^s}{Y}$$

The key second derivatives are

$$\frac{\partial^2 Y}{\partial Z^2} = \frac{(\sigma-1)q\, s^Y_{\ell^s}}{Z}$$
$$\frac{\partial^2 Z}{\partial K^2} = -(1-\rho)\frac{Z_K(1-s^Z_K)}{K}$$
$$\frac{\partial^2 Z}{\partial K\,\partial \ell^c} = (1-\rho)\frac{Z_K(1-s^Z_K)}{\ell^c}$$

where $Z_K \equiv \partial Z/\partial K = \mu K^{\rho-1}Z^{1-\rho}$ and $Z_{\ell^c} \equiv \partial Z/\partial \ell^c = (1-\mu)(\ell^c)^{\rho-1}Z^{1-\rho}$.

Using the chain rule $\partial r/\partial K = Y_{ZZ}Z_K^2 + qZ_{KK}$, the first term is

$$Y_{ZZ}Z_K^2 = \frac{(\sigma-1)qs^Y_{\ell^s}}{Z}\cdot\left(\mu K^{\rho-1}Z^{1-\rho}\right)^2 = \frac{r}{K}(\sigma-1)(1-s^Y_Z)s^Z_K$$

and the second term, using $Z_{KK} = -(1-\rho)(1-s^Z_K)Z_K/K$ and $qZ_K = r$, is

$$qZ_{KK} = -\frac{r}{K}(1-\rho)(1-s^Z_K)$$

giving

$$\frac{dr}{dK} = \frac{r}{K}\left[(\sigma-1)(1-s^Y_Z)s^Z_K - (1-\rho)(1-s^Z_K)\right]$$

For $dw^c/dK = Y_{ZZ}Z_K Z_{\ell^c} + qZ_{K\ell^c}$, the first term is

$$Y_{ZZ}Z_K Z_{\ell^c} = \frac{(\sigma-1)qs^Y_{\ell^s}}{Z}\cdot Z_K\cdot Z_{\ell^c} = \frac{r(1-s^Z_K)}{\ell^c}(\sigma-1)(1-s^Y_Z)$$

and the second term, using $Z_{K\ell^c} = (1-\rho)Z_K Z_{\ell^c}/Z$ and $qZ_{\ell^c} = w^c$, is

$$qZ_{K\ell^c} = (1-\rho)\frac{w^c Z_K}{Z} = \frac{r(1-s^Z_K)}{\ell^c}(1-\rho)$$

where the last step uses $w^c\ell^c s^Z_K = rK(1-s^Z_K)$, giving

$$\frac{dw^c}{dK} = \frac{r(1-s^Z_K)}{\ell^c}\left[(\sigma-1)(1-s^Y_Z) + (1-\rho)\right]$$

Finally, since $w^s = \lambda(\ell^s)^{\sigma-1}Y^{1-\sigma}$ depends on K only through Y,

$$\frac{dw^s}{dK} = \lambda(\ell^s)^{\sigma-1}(1-\sigma)Y^{-\sigma}\cdot r = \frac{(1-\sigma)\,r\,s^Y_{\ell^s}}{\ell^s}$$

Then we have that $\frac{dSW}{dk_t} = \mu_t$. Where μ_t is given by:

$$\mu_t = \Bigg(\sum_{h=1}^{A} \sum_{j} \bigg(\gamma^{t-h} \beta^h \lambda_j \omega_j \mathbb{E}\bigg[u'(c^h_{j,t}) \bigg((1-\tau_{\ell,t}) \theta^h_j \epsilon \frac{dw_{jt}}{dk_t} + (1-\tau_{kt}) \bigg(b^h_{j,t} \frac{dr^b_t}{dk_t} + k^{ch}_{j,t} \frac{dr^c_t}{dk_{j,t}} + \zeta_j k^h_{j,t} \psi \frac{dr_t}{dk_t} \bigg) \bigg) \bigg] + \gamma^t u'(c^0_{j,t})(1-\tau_{\ell t}) \sum_j \theta^0_j \frac{dw_{jt}}{dk_t} \Bigg)$$

Where here, $w_j \equiv \rho_j w^c + (1-\rho_j) w^s$ and ρ_j is the type-j college educated share. Following the same steps as before, we can separate these channels into the impact of risk and redistribution. Define the following constants:

$$\Omega_r \equiv [(\sigma-1)(1-s^Y_z)s^Z_K - (1-\rho)(1-s^Z_K)] \tag{7}$$

$$\Omega_c \equiv (1-s^Z_K)[(\sigma-1)(1-s^Y_Z) + (1-\rho)] \tag{8}$$

$$\Omega_s \equiv (1-\sigma)s^Y_{\ell_s} \tag{9}$$

Then we have that:

$$\mu_t = r_{t+1} \Bigg(\sum_j \omega_j \lambda_j \sum_{h=1}^{A} \gamma^{t-h} \beta^h \mathbb{E}\bigg[u'(c^h_{j,t}) \bigg((\epsilon-1)\theta^h_j \mathcal{M}(1-\tau_{\ell t}) - (1-\tau_k)(\psi-1)\zeta_j \frac{k_{j,t}}{k_t} \bigg) \bigg] + \sum_j \omega_j \lambda_j \sum_{h=1}^{A} \gamma^{t-h} \beta^h \mathbb{E}\Big[u'(c^h_{j,t}) \Big] \bigg(\theta^h_j \mathcal{M}(1-\tau_{\ell t}) - \bigg(\zeta_j \frac{k^h_{j,t}}{k_t} - \frac{k^{ch}_{j,t}}{k_t} - \frac{b^h_{j,t}}{k_t} \bigg)(1-\tau_{kt}) \bigg) + \sum_j \omega_j \lambda_j u'(c^0_{j,t}) \gamma^t (1-\tau_{\ell t}) \theta^0_j \Bigg)$$